Mathematics for Biology

MAT1142

Department of Mathematics
University of Ruhuna

A.W.L. Pubudu Thilan

Probability

Experiment

An **experiment** is a situation involving chance or probability that leads to results called outcomes.

Eg: Tossing a coin.

Outcome

An **outcome** is the result of a single trial of an experiment.

Eg: The possible outcomes are appearing Head or Tail.

Event

An **event** is one or more outcomes of an experiment.

Eg: One event of this experiment is appearing Head.

Probability

Probability is the measure of how likely an event is.

Eg: The probability of appearing Head is 1/2.

Probability of an Event

The probability of event A is the number of ways event A can occur divided by the total number of possible outcomes.

$$p(A) = \frac{\text{The Number Of Ways Event } A \text{ Can Occur}}{\text{The total number Of Possible Outcomes}}$$

Example 1

Suppose we toss a coin and A represent the event of appearing a Head. Find the probability of A.

Solution

$$p(A) = \frac{\text{The Number Of Ways Head Can Occur}}{\text{The total number Of Possible Outcomes}} = \frac{1}{2}$$

Example 2

A single 6-sided die is rolled.

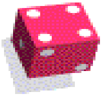
(a) What is the probability of each outcome?

(b) What is the probability of rolling an even number?

(b) What is the probability of rolling an odd number?

Example 2
Solution

(a) The possible outcomes of this experiment are 1, 2, 3, 4, 5 and 6.



$$p(1) = \frac{\text{The number of ways to roll 1}}{\text{Total number of sides}} = \frac{1}{6}$$

$$p(2) = \frac{\text{The number of ways to roll 2}}{\text{Total number of sides}} = \frac{1}{6}$$

$$p(3) = \frac{\text{The number of ways to roll 3}}{\text{Total number of sides}} = \frac{1}{6}$$

$$p(4) = \frac{\text{The number of ways to roll 4}}{\text{Total number of sides}} = \frac{1}{6}$$

$$p(5) = \frac{\text{The number of ways to roll 5}}{\text{Total number of sides}} = \frac{1}{6}$$

$$p(6) = \frac{\text{The number Of ways to roll 6}}{\text{Total number of sides}} = \frac{1}{6}$$

(b) The event of rolling an even number consist of outcomes (2, 4 or 6),

$$p(\text{even}) = \frac{\text{The number of ways to roll an even number}}{\text{Total number of sides}} = \frac{3}{6} = \frac{1}{2}$$

(c) The event of rolling an odd number consist of outcomes (1, 3 or 5),

$$p(\text{odd}) = \frac{\text{The number of ways to roll an odd number}}{\text{Total number of sides}} = \frac{3}{6} = \frac{1}{2}$$

Example 3

A glass jar contains 6 red, 5 green, 8 blue and 3 yellow marbles. If a single marble is chosen at random from the jar, what is the probability of choosing a red marble? a green marble? a blue marble? a yellow marble?

Example 3
Solution

The possible outcomes of this experiment are red, green, blue and yellow.

$$p(\textbf{red}) = \frac{\text{The number of ways to choose red}}{\text{Total number of marbles}} = \frac{6}{22} = \frac{3}{11}$$

$$p(\textbf{green}) = \frac{\text{The number of ways to choose green}}{\text{Total number of marbles}} = \frac{5}{22}$$

$$p(\textbf{blue}) = \frac{\text{The number of ways to choose blue}}{\text{Total number of marbles}} = \frac{8}{22} = \frac{4}{11}$$

$$p(\textbf{yellow}) = \frac{\text{The number of ways to choose yellow}}{\text{Total number of marbles}} = \frac{3}{22}$$

Independent Events

Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

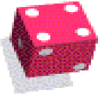
Eg:

- Landing on heads after tossing a coin and rolling a 5 on a single 6-sided die.
- Choosing a marble from a jar and landing on heads after tossing a coin.

When two events, A and B, are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \,.\, P(B)$$

A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

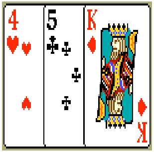


$$\begin{aligned} P(\text{Head}) &= \frac{1}{2} \\ P(3) &= \frac{1}{6} \\ P(\text{head and 3}) &= P(\text{Head}).P(3) \\ &= \frac{1}{2}.\frac{1}{6} \\ &= \frac{1}{12} \end{aligned}$$

Independent Events
Example 2

A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and an eight?

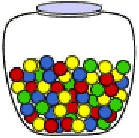
$$
\begin{aligned}
P(\text{Jack}) &= \frac{4}{52} \\
P(8) &= \frac{4}{52} \\
P(\text{Jack and } 8) &= P(\text{Jack}).P(8) \\
&= \frac{4}{52}.\frac{4}{52} \\
&= \frac{16}{2704} \\
&= \frac{1}{169}
\end{aligned}
$$



Independent Events
Example 3

A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and a yellow marble?

$$
\begin{aligned}
P(\text{Green}) &= \frac{5}{16}\\
P(\text{Yellow}) &= \frac{6}{16}\\
P(\text{Green and Yellow}) &= P(\text{Jack}).P(\text{Yellow})\\
&= \frac{5}{16}.\frac{6}{16}\\
&= \frac{30}{256}\\
&= \frac{15}{128}
\end{aligned}
$$



Mutually Exclusive Events

Two events are **mutually exclusive** if they cannot occur at the same time (i.e., they have no outcomes in common).

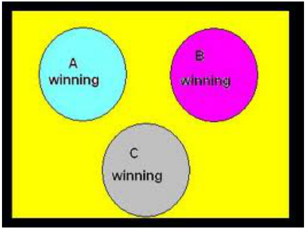


Figure: If teams A, B and C play a game, the event of wining the game is mutually exclusive. Because only one team can win the game.

Mutually Exclusive Events
Example 1

(i) A={1, 3, 5}   B={2, 4, 6}
A and B have no common outcomes. So they are mutually exclusive.

(ii) X={1, 2, 3, 4, 5}   Y={2, 4, 6}
X and Y have common outcomes. So they are non-mutually exclusive.

Mutually Exclusive Events
Example 2

- A single 6-sided die is rolled.
- The number rolled can be an odd number.
- The number rolled can be an even number.
- These events are mutually exclusive since they cannot occur at the same time.

- In the Venn Diagram below, the probabilities of events A and B are represented by two disjoint sets (i.e., they have no elements in common).

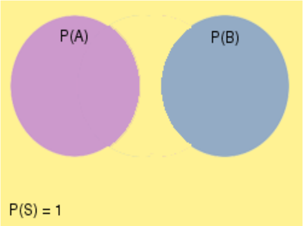


Figure: The Venn Diagram for Mutually Exclusive Events

The Venn Diagram
Non-Mutually Exclusive Events

- In the Venn Diagram below, the probabilities of events A and B are represented by two intersecting sets (i.e., they have some elements in common).

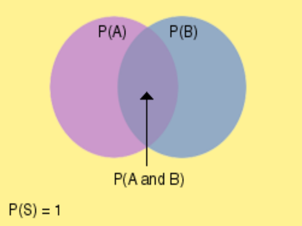


Figure: The Venn Diagram for Non-Mutually Exclusive Events

The Union of Events

(a) Let A and B are two events

$$P(A \cup B) = A(A) + P(B) - P(A \cap B)$$

(b) Let A, B and C are three events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B \cap C)$$

The Union of Mutually Exclusive Events

(a) Let A and B are two mutually exclusive events:

$$P(A\cup B)=P(A)+P(B) \qquad \text{(Since } P(A\cap B)=0\text{)}$$

(b) Let A, B and C are three mutually exclusive events:

$$P(A\cup B\cup C)=P(A)+P(B)+P(C) \qquad \text{(Since } P(A\cap B\cap C)=0\text{)}$$

Example 1

It is known that the probability of obtaining zero defectives in a sample of 40 items is 0.34 whilst the probability of obtaining 1 defective item in the sample is 0.46. What is the probability of

(a) obtaining not more than 1 defective item in a sample?

(b) obtaining more than 1 defective items in a sample?

Example 1
Solution

a) Mutually exclusive, so

$P(E1 \text{ or } E2) = P(E1) + P(E2) = 0.34 + 0.46 = 0.8.$

b) $P(\text{more than } 1) = 1$ - $0.8 = 0.2.$

Example 2

Two persons A and B appeared for an interview for a job. The probability of selection of A is $1/3$ and that of B is $1/2$. Find the probability that:

(i) both of them will be selected.

(ii) only one of them will be selected.

(iii) none of them will be selected.

Example 2
Solution

$$P(A) = \frac{1}{3} \quad P(B) = \frac{1}{2}$$
$$P(\overline{A}) = \frac{2}{3} \quad P(\overline{B}) = \frac{1}{2}$$

Selection or non-selection of any one of the candidate is not affecting the selection of the other. Therefore A and B are independent events.

(i) Probability of selecting both A and B

$$
\begin{aligned}
&= P(A \cap B) \\
&= P(A) \times P(B) \\
&= \frac{1}{3} \times \frac{1}{2} \\
&= \frac{1}{6}
\end{aligned}
$$

Example 2
Solution

(ii) Probability of selecting any one of them

$$
\begin{aligned}
&= P(A \cap \overline{B}) + P(\overline{A} \cap B) \\
&= P(A).P(\overline{B}) + P(\overline{A}).P(B) \\
&= \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \\
&= \frac{3}{6} \\
&= \frac{1}{2}
\end{aligned}
$$

(iii) Probability of not selecting both A and B

$$
\begin{aligned}
&= P(\overline{A} \cap \overline{B}) \\
&= P(\overline{A}) \times P(\overline{B}) \\
&= \frac{2}{3} \times \frac{1}{2} \\
&= \frac{1}{3}
\end{aligned}
$$

Example 3

If the independent probabilities that three people A, B and C will be alive in 30 years time are 0.4, 0.3, 0.2 respectively, calculate the probability that in 30 years time:

a) all will be alive

b) none will be alive

c) only one will be alive

d) at least one will be alive

Solution

a) $P = P(A) \times P(B) \times P(C) = 0.4 \times 0.3 \times 0.2 = 0.024.$

b) We use the notation $P(\overline{A})$ to mean "the probability that A will not occur". So:

$$P = P(\overline{A}) \times P(\overline{B}) \times P(\overline{C}) = 0.6 \times 0.7 \times 0.8 = 0.336.$$

Solution

c)

$$
\begin{aligned}
P &= P(\text{A only alive}) + P(\text{B only alive}) + P(\text{C only alive}) \\
P &= [P(A)\times P(\overline{B})\times P(\overline{C})] + [P(\overline{A})\times P(B)\times P(\overline{C})] \\
&\quad + [P(\overline{A})\times P(\overline{B})\times P(C)]. \\
&= 0.4\times 0.7\times 0.8 + 0.6\times 0.3\times 0.8 + 0.6\times 0.7\times 0.2 \\
&= 0.452
\end{aligned}
$$

d) $P = 1 - [P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})] = 1 - 0.336 = 0.664.$

Thank You