

Mathematical Computing

IMT2b2 β

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Minima and Maxima of Univariate Functions

Introduction

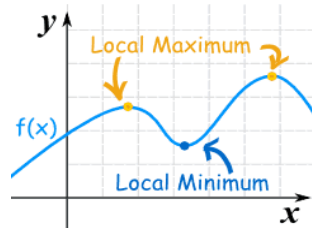
- ▶ When using mathematics to model the physical world in which we live, we frequently express physical quantities in terms of **variables**.
- ▶ Then, **functions** are used to describe the ways in which these variables change.
- ▶ A scientist or engineer will be interested in the ups and downs of a function, its maximum and minimum values, its turning points.

How do we locate maximum and minimum points

- ▶ Drawing a graph of a function using a computer graph plotting package will reveal behaviour of the function.
- ▶ But if we want to know the precise location of maximum and minimum points, we need to turn to algebra and differential calculus.
- ▶ In this section we look at how we can find maximum and minimum points in this way.

Local maximum and local minimum

- ▶ The **local maximum** and **local minimum** (plural: maxima and minima) of a function, are the largest and smallest value that the function takes at a point within a given interval.
- ▶ It may not be the minimum or maximum for the whole function, but **locally** it is.

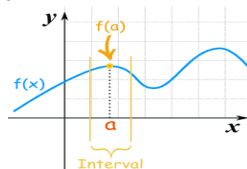


Local maximum and local minimum

Local maximum

- ▶ To define a local maximum, we need to consider an interval.
- ▶ Then a **local maximum** is the point where, the height of the function at **a** is greater than (or equal to) the height anywhere else in that interval.
- ▶ Or, more briefly:

$$f(a) \geq f(x) \text{ for all } x \text{ in the interval.}$$



Local maximum and local minimum

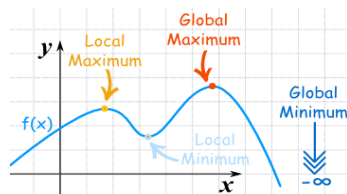
Local minimum

- ▶ To define a local minimum, we need to consider an interval.
- ▶ Then a **local minimum** is the point where, the height of the function at **a** is lowest than (or equal to) the height anywhere else in that interval.
- ▶ Or more briefly:

$$f(a) \leq f(x) \text{ for all } x \text{ in the interval.}$$

Global (or Absolute) Maximum and Minimum

- ▶ The maximum or minimum over the entire function is called an **absolute** or **global** maximum or minimum.
- ▶ There is **only one** global maximum.
- ▶ And also there is **only one** global minimum.
- ▶ But there can be **more than one** local maximum or minimum.



Maxima

Extrema of a univariate function f can be found by the following well-known method:

1. Find the critical points of f , i.e., points a with $f'(a) = 0$.
2. Compute the second derivative f'' and check its sign at these critical points.
 - ▶ If $f''(a) > 0$, then a is a local minimum.
 - ▶ If $f''(a) < 0$, then a is a local maximum.
 - ▶ If $f''(a) = 0$, then we need higher order derivatives at a for a decision.

Maxima

- ▶ It is always a good idea to plot the graph of the function.
- ▶ In order to find all critical points we have to compute the first derivative f' and find all its roots.
- ▶ Next we have to evaluate the second derivative f'' at all these critical points and check the signs of the results.

Thank You