Mathematics for Biology

MAT1142

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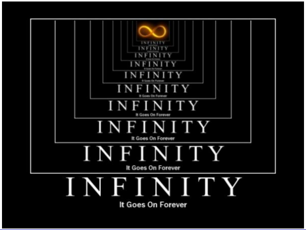
Pre-Requisities for Limits

What is infinity?

- Infinity is the idea of something that has no end.
- Infinity is not a real number, it is an idea.
- That is used to represent something without an end.
- Infinity cannot be measured.
- Infinity is greater than any real number.
- We use symbool ∞ to represent infinity.



Special properties of infinity

1. $\infty + \infty = \infty$
2. $-\infty + (-\infty) = -\infty$
3. $\infty \times \infty = \infty$
4. $-\infty \times -\infty = \infty$
5. $-\infty \times \infty = -\infty$
6. $x + \infty = \infty$
7. $-\infty + x = -\infty$
8. $x - (-\infty) = \infty$

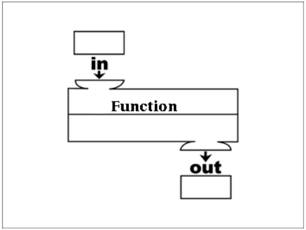
Indeterminate form in Mathematics

- The term "indeterminate" is sometimes used as a synonym for **unknown** or **variable**.
- A mathematical expression can also be said to be indeterminate if it is not definitively or precisely determined.
- There are seven indeterminate forms involving 0, 1, and ∞.
- They are,

$$\frac{\mathbf{0}}{\mathbf{0}}, \mathbf{0} \times \infty, \frac{\infty}{\infty}, \infty - \infty, \mathbf{0}^{\mathbf{0}}, \infty^{\mathbf{0}}, \mathbf{1}^{\infty}.$$

What is function?

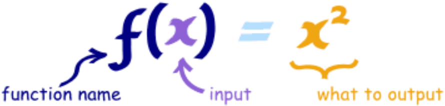
- A function is something like a machine.
- It has an input and an output.
- The output is related somehow to the input.



What is function?

Cont...

- It is useful to give a function a name.
- $f(x)$ is the classic way of writing a function.
- The most common name is f, but you can have other names like $g, h, v, ...$ etc.



Limits

Introduction

- In some situations, we cannot work something out directly.
- But we can see how it behaves as we get closer and closer.
- Let's consider below function as an example:

$$f(x) = \frac{(x^2 - 1)}{(x - 1)}$$

Cont...

- Let's work it out for x=1:

$$f(1) = \frac{(1^2 - 1)}{(1 - 1)} = \frac{0}{0}$$

- We don't really know the value of 0/0.
- So we need another way of answering this.
- The limits can be used to give answer in such a situations.

Cont...

Instead of trying to work it out for x=1, let's try approaching it closer and closer from $x < 1$:

x	$\frac{(x^2-1)}{(x-1)}$
0.5	1.50000
0.9	1.90000
0.99	1.99000
0.999	1.99900
0.9999	1.99990
0.99999	1.99999
...	...

Cont...

Instead of trying to work it out for x=1, let's try approaching it closer and closer from $x > 1$:

x	$\frac{(x^2-1)}{(x-1)}$
1.5	2.50000
1.1	2.10000
1.01	2.01000
1.001	2.00100
1.0001	2.00010
1.00001	2.00001
...	...

Cont...

- Now we can see that as x gets close to 1, then $(x^2 - 1)/(x - 1)$ gets close to 2.
- When $x = 1$ we don't know the answer.
- But we can see that it is going to be 2.

Cont...

- We want to give the answer "2" but can't, so instead mathematicians say exactly what is going on by using the special word "limit".

The limit of $(x^2 - 1)/(x - 1)$ as x approaches 1 is 2

- It can be written symbolicaly as:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2.$$

Properties of limit

Suppose $f(x)$ and $g(x)$ are functions of x and a and c are constants. Then we have following properties for limit.

1. $\lim_{x\to a} c = c$.
2. $\lim_{x\to a} cx = c \lim_{x\to a} x$.
3. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.
4. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$.
5. $\lim_{x\to a} [f(x).g(x)] = (\lim_{x\to a} f(x)) . (\lim_{x\to a} g(x))$.
6. $\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$.

Properties of limit

Examples

Find the following limits:

(i) $\lim_{x \to 2} 5$.

(ii) $\lim_{x \to 3}(4x)$.

(iii) $\lim_{x \to -1}(x + 4)$.

(iv) $\lim_{t \to 2}(t - 1)$.

(v) $\lim_{x \to 2}(x^2)$.

(vi) $\lim_{x \to 4}(1/x)$.

Example 1

Find the limit of $(4-3x)/(3+x)$ as,

(i) $x \to 1$.

(ii) $x \to 4/3$.

(iii) $x \to -3$.

(iv) $x \to 0$.

Example 2

Find the following limits:

(i) $\lim_{x\to 0} \dfrac{x^3 - 7x^2 - 3x}{x}$.

(ii) $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

(iii) $\lim_{x\to 2} \dfrac{x^2 - 4}{x - 2}$.

(iv) $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$.

(v) $\lim_{t\to 1} \dfrac{2 - 2t^2}{t - 1}$.

(vi) $\lim_{t\to -1} \dfrac{2 - 2t^2}{t - 1}$.

Example 3

Find the following limits:

(i) $\lim_{x\to\infty}(5x)$.

(ii) $\lim_{x\to\infty}(5x+7)$.

(iii) $\lim_{x\to\infty}(2x-100)$.

(iv) $\lim_{x\to-\infty}(2x)$.

(v) $\lim_{x\to\infty}\left(\frac{2x}{x+1}\right)$.

(vi) $\lim_{x\to\infty}\left(\frac{x}{2x+1}\right)$.

(vii) $\lim_{x\to\infty}\left(\frac{4x^2+x+1}{3x^2+2x+1}\right)$.

Excercise

Show the followings:

(i) $\lim_{x\to -1}(5x+4) = -1.$

(ii) $\lim_{x\to 1}(5x^2+7x+3) = 15.$

(iii) $\lim_{x\to -1} \dfrac{x^2+5x+3}{2x} = \dfrac{1}{2}.$

(iv) $\lim_{x\to \infty} \dfrac{2x^2+5x+3}{x^2+6x+9} = 2.$

(v) $\lim_{x\to \infty} \left(\dfrac{2x^2+x}{3x^2+4}\right) = \dfrac{2}{3}.$

(vi) $\lim_{x\to 3} \dfrac{x^2-5x+6}{x-3} = 1.$

(vii) $\lim_{x\to 3} \left(\dfrac{x^2-9}{x-3}\right) = 6.$

Thank You