Mathematics for Biology

MAT1142

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Pre-Requisities for Differentiation

Terms and polynomials

- A **term** is an algebraic expression that is either a constant or a product of a constant and one or more variables raised to whole-number powers.
- Examples of terms include: $9x^4,\ 2xy^3,\ 5$.
- A **polynomial** is a finite sum of one or more terms.
- Examples of polynomials are: $x + 12, 8y^4, 6x^2 - 12xy + 9y^2$.

The degree of terms and the degree of polynomials

- The **degree of a term** is the sum of the exponents of its variables.
- The degree of a nonzero constant is zero.
- The **degree of a polynomial** is the highest degree of any of its terms.

The degree of terms and the degree of polynomials

Examples

(a) $2x^5 - 5x^3 - 10x + 9$

This polynomial has four terms, including a fifth-degree term, a third-degree term, a first-degree term, and a constant term. This is a fifth-degree polynomial.

(b) $7x^4 + 6x^2 + x$

This polynomial has three terms, including a fourth-degree term, a second-degree term, and a first-degree term. There is no constant term. This is a fourth-degree polynomial.

General form of a univariate polynomial of degree n

- The simplest polynomials have one variable.
- A one-variable (univariate) polynomial of degree n has the following form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x^1 + a_0 x^0.$$

General form of a univariate polynomial of degree n

Cont...

- Where the a's represent the coefficients and x represents the variable.
- Because $x^1 = x$ and $x^0 = 1$ for all complex numbers x, the above expression can be simplified to:

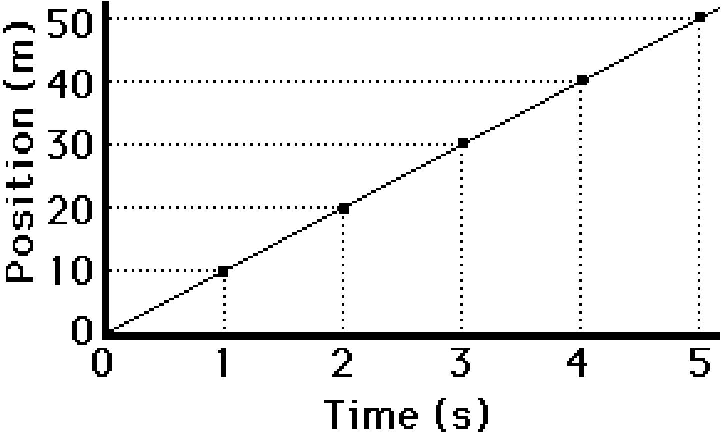
$$a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0.$$

The slope of a graph

- The slope of the line on a position versus time graph is equal to the velocity of the object.
- If the object is moving with a velocity of $+4ms^{-1}$, then the slope of the line will be $+4ms^{-1}$.
- If the object is moving with a velocity of $-8ms^{-1}$, then the slope of the line will be $-8ms^{-1}$.
- If the object has a velocity of $0ms^{-1}$, then the slope of the line will be $0ms^{-1}$.

How to calculate the slope of a line?

Let's begin by considering the position versus time graph below.



How to calculate the slope of a line?

Cont...

- The line is sloping upwards to the right.
- But mathematically, by how much does it slope upwards for every 1 second along the horizontal (time) axis?
- To answer this question we must use the slope equation.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}.$$

Steps need to determine slope

- Pick two points on the line and determine their coordinates.
- Determine the difference in y-coordinates of these two points (rise).
- Determine the difference in x-coordinates for these two points (run).
- Divide the difference in y-coordinates by the difference in x-coordinates (rise/run).

Steps need to determine slope

Example

Find the slop of the line using following points.

(a) For points $(5s, 50m)$ and $(0s, 0m)$.

(b) For points $(5s, 50m)$ and $(2s, 20m)$.

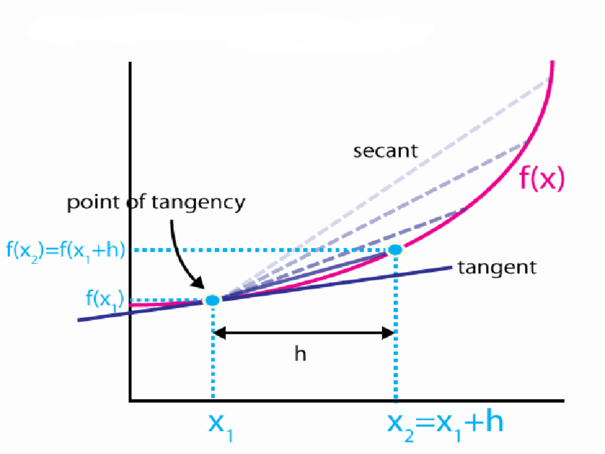
(c) For points $(4s, 40m)$ and $(3s, 30m)$.

How to calculate the slope of a curve?

- To find slope of a curve, we have to consider **tangents** drawn to the couve at different points.
- Tangent means "a straight line that touches a curve at a point but does not intersect it at that point".
- The tangent arises from our desire to find the rate of change of one quantity, $\mathrm{d}y(x)$ say, with respect to another quantity, $\mathrm{d}x$, say, at a given point.

How to calculate the slope of a curve?

Cont...



How to calculate the slope of a curve?

Cont...

- The slope of tangent, denoted $y'(x)$ is a limit of the form:

$$\begin{aligned}
\lim_{h\to 0}\frac{y_2-y_1}{x_2-x_1} &= \lim_{h\to 0}\frac{f(x_2)-f(x_1)}{x_2-x_1}\\
y'(x) &= \lim_{h\to 0}\frac{f(x_1+h)-f(x_1)}{x_1+h-x_1}\\
y'(x) &= \lim_{h\to 0}\frac{f(x_1+h)-f(x_1)}{h}.
\end{aligned}$$

Differentiation

Introduction

- **Differentiation** is the problem of finding the slope of a tangent to a point on a curve.
- Differentiation is concerned with the **rate of change** of one quantity with respect to another quantity.
- Rate of change can generally be expressed as a ratio between a change in one quantity relative to a corresponding change in another quantity.

Derivative

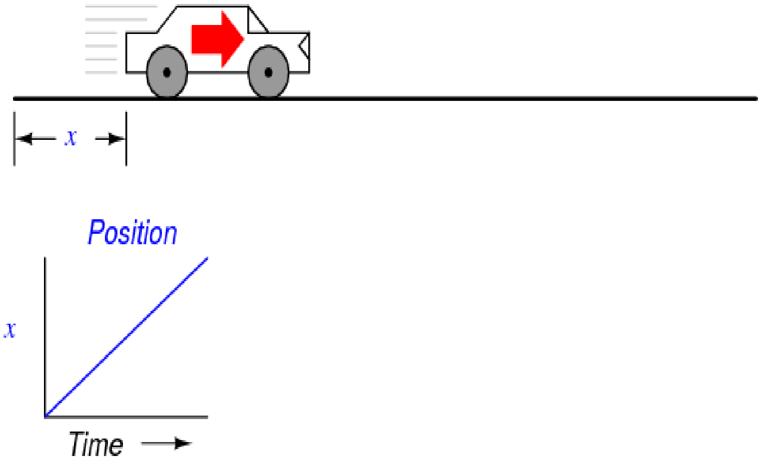
- The instantaneous rate of change of a function $f(t)$ at a point $t = a$ is called the **derivative** of $f(t)$ at $t = a$, denoted by $f'(a)$.

Motivative example 1

- Suppose we were to measure the position of a car, traveling in a direct path (no turns), from its starting point.
- Let us call this measurement, x.
- If the car moves at a rate such that its distance from "start" increases steadily over time, its position will plot on a graph as a linear function (straight line).

Motivative example 1

Position of the car



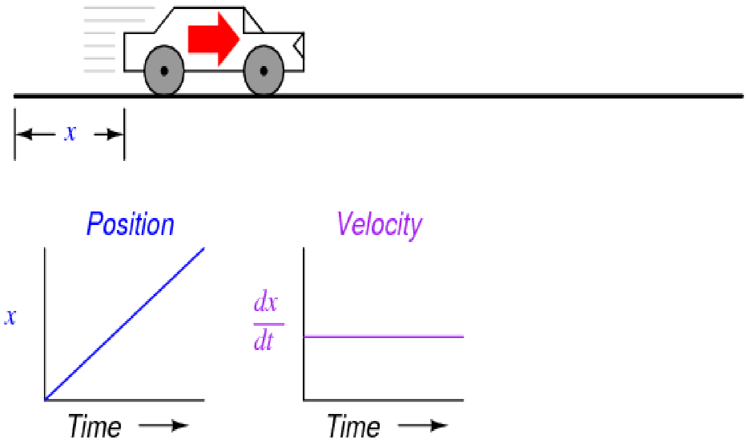
Motivative example 1

Cont...

- The instantaneous rate of change of the car's position with respect to time is called the derivative of the car's position with respect to time.
- The derivative of the car's position with respect to time represents the car's **velocity**.

Motivative example 1

Velocity of the car



Motivative example 1

Cont...

- The instantaneous rate of change of the function $x(t)$ at a point $t = a$ is called the **derivative** of $x(t)$ at $t = a$, denoted by $x'(a)$.
- $x'(a)$ is the velocity of the car at $t = a$.

Motivative example 2

A boy throw a ball vertically upward with a speed of $20ms^{-1}$ then the height of the ball, in metres, after t seconds is approximately,

$$H(t) = 20t - 10t^2.$$

Find the average speed of the ball during the following time intervals.

(a) $t = 0.5s$ to $t = 1s$,

(b) $t = 0.5s$ to $t = 0.75s$,

(c) $t = 0.5s$ to $t = 0.6s$,

(d) $t = 0.5s$ to $t = 0.502s$,

(e) $t = 0.5s$ to $t = 0.501s$,

(f) $t = 0.5s$ to $t = 0.5001s$.

Motivative example 2

Solution

The average speed is given by,

$$\text{Average speed} = \frac{\text{Travelled distance}}{\text{Time taken}}.$$

(a) The average speed from $t = 0.5s$ to $t = 1s$ is:

$$\begin{aligned}
&= \frac{\text{Travelled distance}}{\text{Time taken}} \\
&= \frac{H(1) - H(0.5)}{1 - 0.5} \\
&= \frac{[20 \times 1 - 10 \times (1)^2] - [20 \times 0.5 - 10 \times (0.5)^2]}{1 - 0.5} \\
&= 5ms^{-1}
\end{aligned}$$

Motivative example 2

Solution⇒Cont...

(b) The average speed from $t = 0.5s$ to $t = 0.75s$ is:

$$
\begin{aligned}
&= \frac{\text{Travelled distance}}{\text{Time taken}} \\
&= \frac{H(0.75) - H(0.5)}{0.75 - 0.5} \\
&= \frac{[20 \times 0.75 - 10 \times (0.75)^2] - [20 \times 0.5 - 10 \times (0.5)^2]}{0.75 - 0.5} \\
&= 7.5ms^{-1}
\end{aligned}
$$

Motivative example 2

Solution⇒Cont...

- If we calculate avarage speeds for each of the above time intervals, a good choice for the speed of the ball at $t = 0.5$ is corresponding to the solution of part (f).
- The next example gives the **general solution** to this problem.

Motivative example 3

If, as in motivative example 2, the height of a ball at time t is given by

$$H(t) = 20t - 10t^2,$$

then fnd the following :

(a) the average speed of the ball over the time interval from t to $t + \delta t$,

(b) the limit of this average as $\delta t \to 0$.

Motivative example 3

Solution⇒Cont...

(a) The height at time $t + \delta t$ is $H(t + \delta t)$ and the height at time t is $H(t)$. The difference in heights is $H(t + \delta t) - H(t)$ and the time interval is δt.

$$
\begin{aligned}
H(t+\delta t) - H(t) &= \left[20(t+\delta t) - 10(t+\delta t)^2\right] - \left[20t - 10t^2\right] \\
&= 20\delta t - 20t\delta t - 10(\delta t)^2 \\
&= \delta t\,[20 - 20t - 10\delta t]
\end{aligned}
$$

Motivative example 3

Solution⇒Cont...

The required average speed of the ball from t to $t + \delta t$ is:

$$\begin{aligned} &= \frac{\text{Travelled distance}}{\text{Time taken}} \\ &= \frac{H(t+\delta t) - H(t)}{\delta t} \\ &= \frac{\delta t\,[20 - 20t - 10\delta t]}{\delta t} \\ &= 20 - 20t - 10\delta t. \end{aligned}$$

Motivative example 3

Solution⇒Cont...

(b)

- As δt gets smaller, i.e. $\delta t \to 0$, the last term becomes negligible.
- The instantaneous speed at time t is $20 - 20t$.
- That is the speed of the ball at time t.

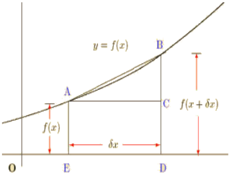
Remark

The speed $v(t)$ is obtained from the height $H(t)$ as,

$$v(t) = \lim_{\delta t \to 0} \left[\frac{H(t + \delta t) - H(t)}{\delta t} \right].$$

The derivative as a limit

- Let $y = f(x)$ is a function as shown in Figure.
- The straight line AB has gradient BC/CA.
- As the point B moves along the curve toward A, the straight line AB tends toward the tangent to the curve at A.
- At the same time, the value of the gradient BC/CA tends toward the gradient of the tangent to the curve at A.



The derivative as a limit

Cont...

$$\frac{BC}{CA} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

The limit, $\mathrm{d}y/\mathrm{d}x$, is called the **derivative** of the function f(x). Its value is the **gradient of the tangent** to the curve at the point $A(x, y)$.

Steps in differentiating a function using definition

1. Let $y = f(x)$.
2. Then $y + \delta y = f(x + \delta x)$.
3. $y + \delta y - y = f(x + \delta x) - f(x) \Rightarrow \delta y = f(x + \delta x) - f(x)$.
4. $\dfrac{\delta y}{\delta x} = \dfrac{f(x + \delta x) - f(x)}{\delta x}$.
5. $\lim_{\delta x \to 0} \dfrac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left(\dfrac{f(x + \delta x) - f(x)}{\delta x} \right)$.
6. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lim_{\delta x \to 0} \left(\dfrac{f(x + \delta x) - f(x)}{\delta x} \right)$.

Rule 1 : The derivative of a constant

The derivative of a constant is zero. That is if $y = c$, where c is a constant, then $\mathrm{d}y/\mathrm{d}x = 0$.

Rule 1 : The derivative of a constant

Examples

Find the derivatives of the followings.

(i) $y = 5.$

(ii) $y = 5a$ where a is a constant.

(iii) $y = \frac{4a}{b^4}$ where a, b are constants.

(iv) $y = a^2b + b^4c + 65c^5$ where a, b, c are constants.

Differentiating functions using definition

Examples

Using basic definition, find the derivatives of the following functions with respect to x.

(i) $y = x$.

(ii) $y = x^2$.

(iii) $y = x^3$.

Differentiating functions using definition

Examples⇒Generalization of results

Function y	Derivative $\mathrm{d}y/\mathrm{d}x$	Can arrange as
x	1	$1x^{1-1}$
x^2	$2x$	$2x^{2-1}$
x^3	$3x^2$	$3x^{3-1}$
x^4	$4x^3$	$4x^{4-1}$
x^5	$5x^4$	$5x^{5-1}$
.	.	.
.	.	.
.	.	.
x^n	nx^{n-1}	nx^{n-1}

Rule 2: The general power rule

If the function is given by $y = x^n$, where n is any positive or negative integer, then:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}.$$

Rule 2: The general power rule

Examples

Find the derivatives of the followings.

(i) $y = x^6$.

(ii) $y = x^{100}$.

(iii) $y = x^{-8}$.

(iv) $y = 1/x^5$.

(v) $y = 1/x^9$.

(vi) $y = 1/x^{-12}$.

Differentiating a constant times a function using definition

Examples

Using basic definition, find the derivatives of the following functions with respect to x.

(i) $y = ax$.

(ii) $y = ax^2$.

(iii) $y = ax^3$.

Differentiating a constant times a function using definition

Examples⇒Generalization of results

Function y	Derivative $\mathrm{d}y/\mathrm{d}x$	Can arrange as
ax	a	ax^{1-1}
ax^2	$2ax$	$2ax^{2-1}$
ax^3	$3ax^2$	$3ax^{3-1}$
ax^4	$4ax^3$	$4ax^{4-1}$
ax^5	$5ax^4$	$5ax^{5-1}$
.	.	.
.	.	.
.	.	.
ax^n	nax^{n-1}	nax^{n-1}

Rule 3: The derivative of a constant times a function

If the function is given by $y = ax^n$, where a is any real number and n is any positive or negative integer, then:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = anx^{n-1}.$$

Rule 3: The derivative of a constant times a function

Examples

Find the derivatives of the followings.

(i) $y = 3x^6$.

(ii) $y = -\frac{1}{4}x^{12}$.

(iii) $y = 4x^{-8}$.

(iv) $y = 4/x^5$.

(v) $y = -18/x^7$.

(vi) $y = \frac{5x^3}{x^{-12}}$.

More examples

Find the derivatives of the following functions.

(i) $y = \sqrt{x}$.

(ii) $y = \dfrac{1}{\sqrt{x}}$.

(iii) $y = \sqrt[7]{x}$.

(iv) $y = \sqrt{x^7}$.

Rule 4: The derivative of a sum or a difference

- If $y = h(x) + g(x)$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x} + \frac{\mathrm{d}g}{\mathrm{d}x}.$$

- If $y = h(x) - g(x)$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x} - \frac{\mathrm{d}g}{\mathrm{d}x}.$$

Rule 4: The derivative of a sum or a difference

Proof

$$
\begin{aligned}
y &= h(x) + g(x) \\
y + \delta y &= h(x + \delta x) + g(x + \delta x) \\
y + \delta y - y &= h(x + \delta x) + g(x + \delta x) - (h(x) + g(x)) \\
\delta y &= h(x + \delta x) + g(x + \delta x) - h(x) - g(x) \\
\delta y &= h(x + \delta x) - h(x) + g(x + \delta x) - g(x)
\end{aligned}
$$

Rule 4: The derivative of a sum or a difference

Proof⇒Cont...

$$\frac{\delta y}{\delta x} = \frac{h(x+\delta x) - h(x) + g(x+\delta x) - g(x)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{h(x+\delta x) - h(x)}{\delta x} + \frac{g(x+\delta x) - g(x)}{\delta x}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{h(x+\delta x) - h(x)}{\delta x} + \lim_{\delta x \to 0} \frac{g(x+\delta x) - g(x)}{\delta x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x} + \frac{\mathrm{d}g}{\mathrm{d}x}$$

Rule 4: The derivative of a sum or a difference

Examples

Find the derivatives of the following functions.

(i) $y = 3x^6 + 5x^4$.

(ii) $y = -\frac{1}{4}x^6 + 3x$.

(iii) $y = 2x^{-6} + 5x^5$.

(iv) $y = \frac{4}{x^5} + \frac{2}{3}$.

(v) $y = -\frac{18}{x^5} - 2x^5$.

(vi) $y = \frac{5x^3}{x^{-12}} + x^{1/3}$.

Rule 5: The derivative of a polynomial function

- Let $y = f(x)$ be a polynomial function.
- $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} = f'(x)$.
- $f'(x)$ is called the derivative of the polynomial $f(x)$.

Rule 5: The derivative of a polynomial function

The derivative of a univariate polynomial of degree n

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x) = a_n n x^{n-1} + a_{n-1}(n-1)x^{(n-1)-1} + ... + a_2 2x^{2-1} + a_1 x^{1-1} + 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x) = a_n n x^{n-1} + a_{n-1}(n-1)x^{n-2} + ... + a_2 2x + a_1.$$

Rule 5: The derivative of a polynomial function

Examples

Find the derivatives of the following functions.

(i) $y = 2x^{-6} + 5x^4 + 9x + 8.$

(ii) $y = 9 + \frac{2}{x} - \frac{5}{x^2}.$

(iii) $y = \frac{1}{3}x^3 + \frac{1}{7}x + \frac{1}{5}.$

(iv) $y = 2t^2 + 6t + 9.$

(v) $v = 5x^n - nx^7 + 8n.$

(vi) $h = u^2 + \frac{1}{u^2} + 9u + 5.$

Rule 6: The derivatives of trigonometric functions

y	$\frac{\mathrm{d}y}{\mathrm{d}x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Rule 6: The derivatives of trigonometric functions

Examples

Find the derivatives of the following functions.

(i) $y = 2x^3 + \sin x$.

(ii) $y = \tan x + \cos x + x^4 + 7$.

(iii) $y = \cot x + \dfrac{1}{x^5}$.

(iv) $y = \cos t + t^4 + 7t + \dfrac{1}{t^5}$.

(v) $v = x + \dfrac{1}{\sin x}$.

(vi) $h = \sin x \cot x + x \csc x \sin x$.

Rule 7: The derivative of exponential function

$$
\begin{aligned}
y &= e^x \\
y &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \frac{5x^4}{5!} + \ldots \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= 0 + 1 + \frac{2x}{1 \times 2} + \frac{3x^2}{1 \times 2 \times 3} + \frac{4x^3}{1 \times 2 \times 3 \times 4} \\
&+ \frac{5x^4}{1 \times 2 \times 3 \times 4 \times 5} + \ldots \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= e^x
\end{aligned}
$$

Rule 7: The derivative of exponential function

Examples

Find the derivatives of the following functions.

(i) $y = 2e^x + 5x$.

(ii) $y = \tan x + \cos x + x^4 + 7e^x$.

(iii) $y = \dfrac{e^x}{6} + \dfrac{1}{x^4}$.

(iv) $y = 5e^t + t^6 + 7(t + e^t)$.

(v) $v = x + \dfrac{1}{e^{-x}}$.

(vi) $h = \dfrac{1}{e^{-x}} + \dfrac{1}{\tan x}$.

Rule 8: The derivative of a product

The derivative of the product $y = u(x)v(x)$, where u and v are both functions of x is,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = u \times \frac{\mathrm{d}v}{\mathrm{d}x} + v \times \frac{\mathrm{d}u}{\mathrm{d}x}.$$

Rule 8: The derivative of a product

Examples

Find the derivatives of the following functions.

(i) $y = (x + 1)(x + 2)$.

(ii) $y = x^3 \sin x$.

(iii) $y = \sin x \cos x$.

(iv) $y = e^x(3x^5 - 1)$.

(v) $p = (x^4 + 2x)(x^5 - 8x)$.

(vi) $h = \sin^2 x \csc x \tan x$.

Rule 9: The derivative of a quotient

The derivative of the quotient $y = u(x)/v(x)$, where u and v are both function of x is:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v \times \dfrac{\mathrm{d}u}{\mathrm{d}x} - u \times \dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}.$$

Rule 9: The derivative of a quotient

Examples

Find the derivatives of the following functions.

(i) $y = \dfrac{(x+1)}{(x+2)}$.

(ii) $y = \dfrac{(x^3-2)}{2x^2}$.

(iii) $y = \dfrac{x^3+5x-4}{x^2-2}$.

(iv) $y = \dfrac{\sin t + t}{\cos t}$.

(v) $p = \dfrac{e^x}{\cos x}$.

(vi) $h = \dfrac{(x^2+1)}{e^x - \tan x}$.

Rule 10: The derivative of a function of function

If y is a function of u, i.e. $y = f(u)$, and u is a function of x, i.e. $u = g(x)$ then the derivative of y with respect to x is:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x} \text{ (The chain rule).}$$

Rule 10: The derivative of a function of function

Examples

Find the derivatives of the following functions.

(i) $y = (x + 2)^3$.

(ii) $y = (5x^5 - 9x^2 - 4x + 9)^7$.

(iii) $y = (3x^4 + 6)^3$.

Short way for chain rule

Examples

Find the derivatives of the following functions.

(i) $y = (x+2)^3$.

(ii) $y = (5x^5 - 9x^2 - 4x + 9)^7$.

(iii) $y = (3x^4 + 6)^3$.

(iv) $y = \sqrt{1 + x^2}$.

(v) $y = \sin(x^2)$.

(vi) $y = \cos(x^2 + 4x + 3)$.

(vii) $y = e^{2x}$.

(viii) $y = e^{x^2}$.

(ix) $y = e^{x^3+4x+5}$.

(x) $y = \dfrac{1}{\sqrt{x^2+1}}$.

Mixed examples

Find the derivatives of the following functions.

(i) $y = (x+2)^3$.

(ii) $y = (4x^3 - 13x^2 - 4x + 9)^6$.

(iii) $y = (2x^3 + 6)^4$.

(iv) $y = \sqrt{1 + x^2}$.

(v) $y = (x^2 + \cos x)^5$.

(vi) $y = (x^3 + \tan(x^2 + 2))^3$.

(vii) $y = \cos \sqrt{x^2 + 1}$.

(viii) $y = \dfrac{e^{(x^3 + 3x - 5)}}{\sin(x^3 - 2)}$.

Mixed exercise

(i) $y = 5x^2 + 9x^4$.

(ii) $y = 4x^2 + \dfrac{1}{x^2}$.

(iii) $y = 5\sqrt{x} + \dfrac{3}{x^4} - 7x^2$.

(iv) $y = 2\sqrt{x}$.

(v) $y = 4x^{-3} - 2\sin x$.

(vi) $y = 3x^{1/3} + 4x^{-1/4}$.

(vii) $y = (7x^2 + 2x)(x^3 + 1)$.

(viii) $y = \dfrac{x^2 + 8}{2x - 1}$.

(ix) $y = \dfrac{x\cos x}{\sin(2x + 1)}$.

(x) $y = (x^2 - 6)^4$.

(xi) $y = e^{x^3 + 4x + 8}$.

(xii) $y = \sin(4x + 5)$.

(xiii) $y = \tan(x^2 + 1)$.

(xiv) $y = \cos(t^3 + 4t + 5)$.

Thank You