Mathematical Computing

IMT2b2β

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Minima and Maxima of Univariate Functions

Introduction

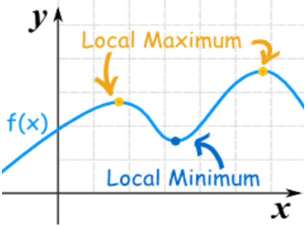
- When using mathematics to model the physical world in which we live, we frequently express physical quantities in terms of **variables**.
- Then, **functions** are used to describe the ways in which these variables change.
- A scientist or engineer will be interested in the ups and downs of a function, its maximum and minimum values, its turning points.

How do we locate maximum and minimum points?

- Drawing a graph of a function using a computer graph plotting package will reveal behaviour of the function.
- But if we want to know the precise location of maximum and minimum points, we need to turn to algebra and differential calculus.
- In this section we look at how we can find maximum and minimum points in this way.

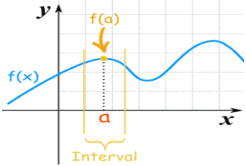
Local maximum and local minimum

- The **local maximum** and **local minimum** (plural: maxima and minima) of a function, are the largest and smallest value that the function takes at a point within a given interval.
- It may not be the minimum or maximum for the whole function, but **locally** it is.



- To define a local maximum, we need to consider an interval.
- Then a **local maximum** is the point where, the height of the function at **a** is greater than (or equal to) the height anywhere else in that interval.
- Or, more briefly:

$$\mathbf{f(a) \geq f(x) \text{ for all } x \text{ in the interval}}.$$

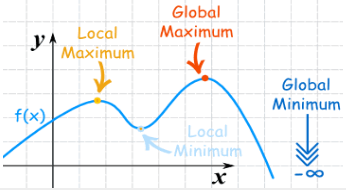


- To define a local minimum, we need to consider an interval.
- Then a **local minimum** is the point where, the height of the function at **a** is lowest than (or equal to) the height anywhere else in that interval.
- Or more briefly:

$$\mathbf{f(a) \leq f(x) \text{ for all } x \text{ in the interval}}.$$

Global (or Absolute) Maximum and Minimum

- The maximum or minimum over the entire function is called an **absolute** or **global** maximum or minimum.
- There is **only one** global maximum.
- And also there is **only one** global minimum.
- But there can be **more than one** local maximum or minimum.



How to classify stationay points?

Extrema of a univariate function $\mathbf{f}$ can be found by the following well-known method:

1. Find the critical points of $\mathbf{f}$, i.e., points $\mathbf{a}$ with $\mathbf{f}'(\mathbf{a}) = \mathbf{0}$.
2. Compute the second derivative $\mathbf{f}''$ and check its sign at these critical points.
   - If $\mathbf{f}''(\mathbf{a}) > 0$, then $\mathbf{a}$ is a local minimum.
   - If $\mathbf{f}''(\mathbf{a}) < 0$, then $\mathbf{a}$ is a local maximum.
   - If $\mathbf{f}''(\mathbf{a}) = 0$, then we need higher order derivatives at $\mathbf{a}$ for a decision.

How to use to Maxima to classify stationay points?

- It is always a good idea to plot the graph of the function.
- In order to find all critical points we have to compute the first derivative $\mathbf{f}'$ and find all its roots.
- Next we have to evaluate the second derivative $\mathbf{f}''$ at all these critical points and check the signs of the results.

Example

Find the stationary points of $f(x) = x^4 - 3x^2 + 2$ and determine the nature of these points.

Thank You