Mathematical Computing
IMT2b2β

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Integration

Indefinite integral

- The command, **integrate(expr, var)** can be used to find indefinite integral of **expr** with respect to variable **var**.

- $\int$ **expr** d(**var**) $\Rightarrow$ **integrate(expr, var)**.

- The indefinite integral returned by **integrate** does not include the arbitrary constant of integration.

- If **integrate** does not succeed, then the return value is the noun form of the integral or an expression containing one or more noun forms.

- The noun form is displayed with an integral sign.

Indefinite integral
Examples

(i) $\int x^2 \, \mathrm{d}x$

(ii) $\int (x^4 + 2x + 4) \, \mathrm{d}x$

(iii) $\int \sqrt{x} \, \mathrm{d}x$

(iv) $\int \sin x \cos x \, \mathrm{d}x$

(v) $\int e^{2x} \, \mathrm{d}x$

(vi) $\int e^{t^2} \, \mathrm{d}t$

(vii) $\int \sin^3 x \, \mathrm{d}x$

(viii) $\int (b^2 - x^2)^{-1/2} \, \mathrm{d}x$

(ix) $\int \tan(\ln(x)) \, \mathrm{d}x$

(x) $\int \cos(\ln(u)) \, \mathrm{d}u$

Definite integral

- The command, **integrate(expr, var, a, b)** can be used to find definite integral of **expr** with respect to variable **var**, with limits of integration **a** and **b**.

- $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{expr}\ \mathrm{d}(\mathbf{var}) \Rightarrow$ **integrate(expr, var, a, b)**.

- If integrate does not succeed, then the return value is the noun form of the integral or an expression containing one or more noun forms.

- The noun form is displayed with an integral sign.

Definite integral
Examples

(i) $\displaystyle\int_1^3 x^3 \ \mathrm{d}x$

(ii) $\displaystyle\int_0^2 (x^6 + 8x + 9) \ \mathrm{d}x$

(iii) $\displaystyle\int_0^\pi \sin x \cos x \ \mathrm{d}x$

(iv) $\displaystyle\int_{-2}^2 e^{t^2} \ \mathrm{d}t$

(v) $\displaystyle\int_1^2 \tan(\ln(x)) \ \mathrm{d}x$

(vi) $\displaystyle\int_{-1}^1 \sin(\ln(u)) \ \mathrm{d}u$

Multiple integrals

- Multiple integrals are evaluated using multiple calls to the **integrate()** function.

- To evaluate double integrals, **integrate()** function should be called two times.

- $\displaystyle\int_c^d \int_a^b$ **expr** $\mathrm{d}x\mathrm{d}y \Rightarrow$ **integrate(integrate(expr,x,a,b),y,c,d)**.

Multiple integrals
Examples

Evaluate the following double integrals.

(i) $\displaystyle\int_0^3 \int_1^2 x^2 y \mathrm{d}y \mathrm{d}x$

(ii) $\displaystyle\int_1^2 \int_0^3 x^2 y \mathrm{d}x \mathrm{d}y$

(iii) $\displaystyle\int_1^3 \int_0^1 (1 - 4xy) \mathrm{d}x \mathrm{d}y$

(iv) $\displaystyle\int_0^2 \int_0^{\pi/2} x \sin y \mathrm{d}y \mathrm{d}x$

Partial Fractions

Partial fractions

- The command **partfrac(expr,var)**, can be used to get partial fractions of a given expression **expr** with respect to variable **var**.

- If there is more than one variable, the variable of interest must be specified.

Partial fractions
Examples

Find partial fractions of following expressions.

(i) $\dfrac{3x + 5}{x^2 + 3x + 2}$

(ii) $\dfrac{3x + 1}{(x + 1)^2}$

(iii) $\dfrac{4x - 1}{x^2 - 4x + 4}$

(iv) $\dfrac{(x - 1)}{(x + 1)(x^2 + 1)}$

(v) $\dfrac{3n^2 + 2n}{n^3 - 3n^2 + 2n - 6}$

(vi) $\dfrac{2xy^3 - 2y^3 + 6x^2y^2 + 5xy^2 - 4x^3y - 2x^2y - 8x^4}{xy^3 - x^2y^2 - 2x^3y}$

Limits

Both side limits

- The command, **limit(expr,x,a)** computes the limit of a given expression **expr** when variable **x** approaches **a**.

- $\lim_{x \to a} \mathbf{expr} \Rightarrow \mathbf{limit(expr,x,a)}$.

- It requires an expression, name of the variable with the limit point to approach.

Both side limits
Examples

(i) $\lim_{x \to 2} 5$

(ii) $\lim_{x \to 3} (4x)$

(iii) $\lim_{x \to -1} (x + 4)$

(iv) $\lim_{t \to 2} (t - 1)$

(v) $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}$

(vi) $\lim_{x \to 4} (1/x)$

One side limits

- It is also possible to compute limits from above and below by adding a forth argument.

- It may have the respective values **plus** for limits from above and **minus** for limits from below.

- $\lim_{x \to a^+}$ **expr** $\Rightarrow$ **limit(expr,x,a,plus)**.

- $\lim_{x \to a^-}$ **expr** $\Rightarrow$ **limit(expr,x,a,minus)**.

One side limits
Examples

(i) $\lim_{x \to 0^+} \ln x$

(ii) $\lim_{x \to 0^+} \dfrac{x}{|x|}$

(iii) $\lim_{x \to 0^-} \dfrac{x}{|x|}$

(iv) $\lim_{x \to 0} \dfrac{x}{|x|}$

Thank You