Mathematical Computing

IMT2b2β

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Differentiation

First derivative

- Maxima can compute derivatives of a given function.
- It provides a single command **diff** for this purpose.
- The command **diff(expr,x)**, differentiates the expression **expr** with respect to variable **x**.

First derivative

Examples

Find first derivative of the following functions.

(i) $y = 3x^6 + 5x^4$.

(ii) $y = -\frac{1}{4}x^6 + 3x$.

(iii) $y = 2x^{-6} + 5x^5$.

(iv) $y = \frac{4}{x^5} + \frac{2}{3}$.

(v) $y = -\frac{18}{x^5} - 2x^5$.

(vi) $y = e^{x^2+2x+5}$.

Derivatives of higher order

- Maxima can compute derivatives of higher order for a given function.
- To compute higher derivatives we have to use command **diff** with additional argument.
- **diff(expr,x,n)** gives **n**-th derivative of **expr** with respect to **x**.

Derivatives of higher order

Examples

Find second and third derivatives of the following functions.

(i) $y = x^3$.

(ii) $y = \dfrac{(x^3 - 2)}{2x^2}$.

(iii) $y = \dfrac{x^3 + 5x - 4}{x^2 - 2}$.

(iv) $y = \dfrac{\sin t + t}{\cos t}$.

(v) $v = \dfrac{e^x}{\cos x}$.

(vi) $y = \dfrac{(x^2 + 5x + 6)}{e^{2x} - \sin x}$.

Partial derivatives

- A partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant.
- Maxima can also compute partial derivatives.
- The command **diff(f(x,y,z),x,1)** gives the partial derivative of **f(x,y,z)** with respect to **x**.
- Similarly, the command **diff(f(x,y,z),y,1)** gives the partial derivative of **f(x,y,z)** with respect to **y**.
- In here 1 indicates first partial derivative.

Partial derivatives

Examples

Use Maxima to find the first order partial derivatives with respect to x and y of the following functions.

(i) $f(x, y) = x^3y + y^3x$.

(ii) $g(x, y) = \sin(x + y^2)$.

(iii) $h(x, y) = \ln(x^2 + y^4 + 1)$.

(iv) $f(x, y) = xe^{xy^2}$.

Higher order partial derivatives

- To calculate the second-order partial derivative of **f(x,y,z)** with respect to **x** and **y**, we use **diff(f(x,y,z),x,1,y,1)**.
- To calculate the third-order partial derivative of **f(x,y,z)** with respect to **x**, **x**, **z** we use **diff(f(x,y,z),x,2,z,1)** and so on.

Higher order partial derivatives

Examples

(i) Find the second order partial derivative of xe^{xy^2} with respect to x and y.

(ii) Find the third order partial derivative of $\ln(x + y^3 + z^5)$ with respect to x, y and z.

The total differential

- When we call **diff** with only one argument (i.e., without passing any variable names), then Maxima computes the total differential of the given expression.
- The differentials of the variables are represented by symbol **del**.
- We can replace **del(x)** and **del(y)** by **dx** and **dy**.

The total differential

Examples

$$\Rightarrow \quad diff(x^\wedge 2 * y + y^\wedge 2 * x);$$

$$\Rightarrow \quad (2 * x * y + x^\wedge 2) * del(y) + (y^\wedge 2 + 2 * x * y) * del(x)$$

$$\Rightarrow \quad \%, del(x) = dx, del(y) = dy;$$

$$\Rightarrow \quad (2 * x * y + x^\wedge 2) * dy + (y^\wedge 2 + 2 * x * y) * dx$$

The gradient

- The gradient of a single-valued function can be computed as Jacobian.
- Notice that the function must be given as a list with one element.
- The command **jacobian(f,x)** gives the gradient of single-valued function **f** with respect to vector **x**.

The gradient

Examples

Find the gradient of following functions.

(i) $f(x, y) = x^2 - 2xy + 6x - 2y + 1$.

(ii) $g(x, y) = e^{xy}$.

Jacobian matrix

- The Jacobian matrix is the matrix of all first-order partial derivatives of a vector- or scalar-valued function with respect to another vector.
- The command **jacobian(f,x)** gives the Jacobian matrix of vector-valued function **f** with respect to vector **x**.
- In Maxima Vector-values functions are represented by vectors of (single-values) functions.
- The derivative of such functions is called the **Jacobian matrix**.

Jacobian matrix

Examples

Find the Jacobian matrix of following functions.

(i) $f(x, y) = (x^2 - 2xy + 6x - 2y + 1, x^3y + y^2x^3)$.

(ii) $g(x, y) = (e^{xy}, \sin(xy))$.

Thank You