Mathematical Computing

IMT2b2β

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Solving Equations

Introduction

- Maxima has several functions which can be used for solving sets of algebraic equations and for finding the roots of an expression.
- Maxima's ability to solve equations is limited, but progress is being made in this area.

The function *solve*()

- The function **solve** can be used to solves a system of simultaneous linear or nonlinear polynomial equations for the specified variable(s) and returns a list of the solutions.
- The Maxima manual has an extensive entry for the important function **solve**.

The function *solve*()

Syntax to solve one equation

- The Maxima syntax to solve one equation is:

  solve(expr, x)
  solve(expr)

- It solves the algebraic equation **expr** for the variable **x** and returns a list of solution equations in **x**.
- If **expr** is not an equation **expr=0** is assumed in its place.
- **x** may be omitted if **expr** contains only one variable.

The function *solve*()

Syntax to solve one equation⇒Different ways to give arguments

- It is important to recognise that the first argument to **solve** is either an equation such as $\mathbf{f(x) = g(x)}$ (or $\mathbf{h(x) = 0}$), or simply $\mathbf{h(x)}$.

- If you just give $\mathbf{h(x)}$, the command **solve** understands that you mean the equation $\mathbf{h(x) = 0}$, and the problem is to find the roots of $\mathbf{h(x)}$.

- The roots of $\mathbf{h(x)}$ means, values of $\mathbf{x}$ such that the equation $\mathbf{h(x) = 0}$ is satisfied.

The function *solve*()

Syntax to solve one equation⇒Different ways to give arguments⇒Try followings

1. (i) $solve(x^\wedge 2+3*x-1=0,x);$
   (ii) $solve(x^\wedge 2+3*x-1=0);$
   (iii) $solve(x^\wedge 2+3*x-1,x);$
   (iv) $solve(x^\wedge 2+3*x-1);$

2. (i) $solve(2*x-4=0,x);$
   (ii) $solve(2*x-4=0);$
   (iii) $solve(2*x-4,x);$
   (iv) $solve(2*x-4);$

The function *solve*()

Syntax to solve one equation⇒Different ways to give arguments⇒Exercise

(i) $x - 6 = 8$

(ii) $2x - 4 = -15$

(iii) $\sqrt{x - 10} - 4 = 0$

(iv) $x^2 + 5x + 3 = 0$

(v) $t^2 - t + 6 = 0$

(vi) $\dfrac{1}{(x-3)} + \dfrac{1}{(x+3)} = \dfrac{10}{(x^2-9)}$

The function *solve*()

Further computations on solutions

- We can assign equations to variables.
- The solutions can be reverted into "non-equations" or used in further computations by means of command **ev**.
- An alternative way is to use command **rhs** to extract the expression from the right-hand side of the equation.

The function *solve*()

Further computations on solutions⇒Try followings

1. (i) $\Rightarrow sol : solve(x^\wedge 2 + 3 * x - 1, x);$
   (ii) $\Rightarrow ev(x, sol[1]);$
   (iii) $\Rightarrow rhs(sol[2]);$

2. (i) $\Rightarrow eq : solve(2 * x^\wedge 2 - 5 * x + 1 = 0, x);$
   (ii) $\Rightarrow ev(x, eq[2]);$
   (iii) $\Rightarrow rhs(eq[1]);$

The function *solve*()

Systems of two or more expressions

- Systems of two or more expressions as well as their variables must be encapsulated in lists.
- Each solution is then also returned as list.

The function *solve*()

Systems of two or more expressions⇒Try followings

1. (i) $\Rightarrow eq1 : 3 * x^{\wedge}2 - y^{\wedge}2 = 2;$
   (ii) $\Rightarrow eq2 : x^{\wedge}2 + y^{\wedge}2 = 2;$
   (iii) $\Rightarrow solve([eq1, eq2], [x, y]);$

2. (i) $\Rightarrow [4 * x^{\wedge}2 - y^{\wedge}2 = 12, x * y - x = 2];$
   (ii) $\Rightarrow solve(\%, [x, y]);$

The function *solve*()

Systems of two or more expressions⇒Exercise

1. Solve the following systems of equations:

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 2 \\ 4x_1 + 3x_2 + x_3 &= 10 \\ x_1 + 2x_2 + 4x_3 &= 5 \end{aligned}$$

The function *allroots*(*expr*)

- In general there are no closed form solutions for the roots of polynomials of degree 5 or larger.
- The **allroots** computes numerical approximations of the real and complex roots of the polynomial equation of one variable.

The function *allroots*(*expr*)

Try followings

1. $\Rightarrow solve(x^{\wedge}5 - x^{\wedge}4 + 2 * x^{\wedge}3 + x^{\wedge}2 - x + 5, x);$
2. $\Rightarrow allroots(x^{\wedge}5 - x^{\wedge}4 + 2 * x^{\wedge}3 + x^{\wedge}2 - x + 5);$

Thank You