Mathematics for Biology

MAT1142

Department of Mathematics
University of Ruhuna

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About course unit

Subject	Mathematics for Biology
Course unit	MAT1142
Number of lecture hours	30hrs
Credit value	2 (Not counted for the Degree)
Method of assessment	End of semester examination

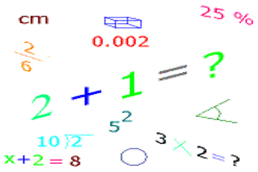
Reference and course materials

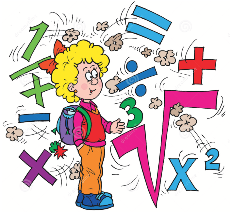
1. **Maths. A self Study Guide** by Jenny Olive. Second Edition. (510 OLI).
2. **Pure Mathematics I** by L Bostock and S. Chandler.
3. **Basic concepts of elementary mathematics** by Peterson, John M. (510PET).
4. **A biologist's basic mathematics** by Causton, David R. (510.24574CAU).
5. www.math.ruh.ac.lk/$\sim$pubudu

What is mathematics?

What is mathematics?

- Many people consider mathematics as doing computations such as addition, subtraction and multiplication of numbers.
- But that is not true.
- It is just part of mathematics and mathematics has great variety of applications in the real world.





Mathematics in our day to day life

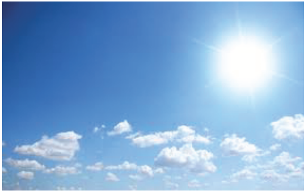
- We use mathematics for such simple tasks as telling the time from a clock or counting our change after making a purchase.
- We also use mathematics for more difficult tasks, such as, making up a household budget.
- Cooking, driving, gardening, sewing and many other common activities often require mathematical calculations involving measurement.
- Mathematics is also part of physics, medicine, computer science, and even sports.

Mathematics in physics

- The mass of the Earth is 5.98×10^{24} kg.
- How do we find it?
- Can we use a balance?

Mathematics in physics
Cont...

- The temperature at the very center of the Sun is about 15,000,000 degrees Celsius.
- How do we measure it?
- Can we use a thermometer?

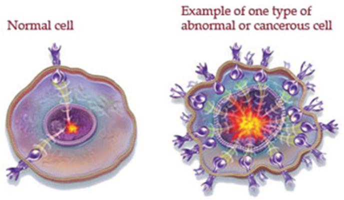


Mathematics in medicine

- A typical adult has a blood volume of approximately between 4.7 and 5 litres.
- How do we measure it?
- Can we find a person for this?

Mathematics in medicine
Cont...

- Clustering algorithms have widely been used in microarray data analysis.
- Those algorithms can be used to identify cancer tissues.



Mathematics in computer science

- Usually we have password for protecting our personal computers.
- So, at the time of loging, a user has to give his password for the varification.
- But now a user can use his face as a password.
- This technology is called as face recognition and it has been developed completely based on mathematics.

- **WinZip** is a Windows program that lets you compress files so that you can store or distribute them more efficiently.
- The theoretical background of compression is provided by **information theory** for lossless compression and **rate-distortion theory** for lossy compression.
- The idea of data compression is deeply connected with **statistical inference**.

Mathematics in sport

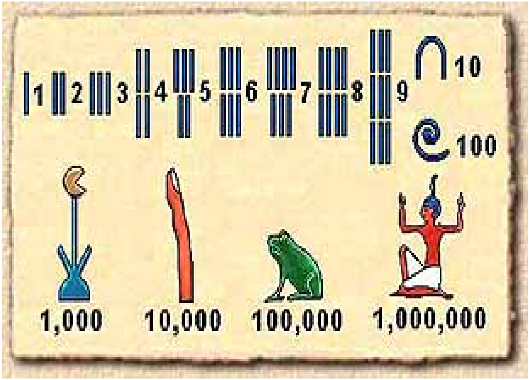
- The Duckworth-Lewis method (D/L method) is a mathematical formulation designed to calculate the target score for the team batting second in a limited overs match interrupted by weather or other circumstances.
- It is generally accepted to be the most accurate method of setting a target score.
- The D/L method was devised by two English statisticians, Frank Duckworth and Tony Lewis.

Basic Algebra

The development of numbers

- The numbers were used even by people in the Stone Age for counting things like property and enemies.
- The Egyptians were the first civilization to invent different symbols for different numbers.

Ancient Egyptian numbers



Different symbols used to represent numbers

- 1 is shown by a single stroke.
- 10 is shown by a drawing of a hobble for cattle.
- 100 is represented by a coil of rope.
- 1,000 is a drawing of a lotus plant.
- 10,000 is represented by a finger.
- 100,000 by a tadpole or frog.
- 1,000,000 is the figure of a god with arms raised above his head.

Reading and writing of large numbers

- The conventions for reading and writing numbers is quite simple.
- The higher number is always written in front of the lower number.

Use of numbers in today

- A number is a mathematical object used in counting and measuring.
- Different types of numbers are used in different cases.
- Numbers can be classified into sets, called number systems.

Different type of numbers

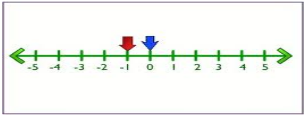
- Natural numbers ($\mathbb{N}$)
- Integers ($\mathbb{Z}$)
- Rational numbers ($\mathbb{Q}$)
- Irrational numbers ($\mathbb{Q}^c$)
- Real numbers ($\mathbb{R}$)
- Complex numbers ($\mathbb{C}$)

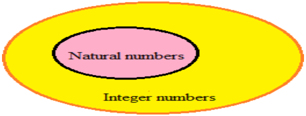
Natural numbers ($\mathbb{N}$)

- The natural numbers had their origins in the words used to count things (Eg: Five oranges, two boys etc).
- Zero was not even considered a number for the Ancient Greeks.
- There is no universal agreement about whether to include zero in the set of natural numbers.
- Some define the natural numbers to be $\{1, 2, 3, ...\}$, while others define it as $\{0, 1, 2, 3, ...\}$.

Integers ($\mathbb{Z}$)

- The **integers** are formed by the natural numbers (including 0) (0, 1, 2, 3, ...) together with the negatives of the non-zero natural numbers (-1, -2, -3, ...).
- **Positive integers** are all the whole numbers greater than zero: 1, 2, 3, 4, 5, ...
- **Negative integers** are all the opposites of these whole numbers: -1, -2, -3, -4, -5, ...
- We do not consider zero to be a positive or negative number.





Rational numbers ($\mathbb{Q}$)

- A rational number is a number that can be expressed as a fraction with an integer numerator and a non-zero integer number denominator.

$$\mathbb{Q} = \{r = \frac{p}{q} | p, q \in \mathbb{Z}; q \neq 0\}$$

Check whether following numbers are rational or not.

Number	As a Fraction	Rational?
5		
1.75		
.001		
0.111...		
$\sqrt{2}$		

Rational numbers ($\mathbb{Q}$)

Example $\Rightarrow$ Solution

Number	As a Fraction	Rational?
5	$5/1$	Yes
1.75	$7/4$	Yes
.001	$1/1000$	Yes
0.111...	$1/9$	Yes
$\sqrt{2}$	?	NO

Irrational numbers ($\mathbb{Q}^c$)

- Some numbers cannot be written as a ratio of two integers.
- They are called as irrational numbers.
- It is called irrational because it cannot be written as a ratio (or fraction).

Irrational numbers ($\mathbb{Q}^c$)
Example 1

π is an irrational number

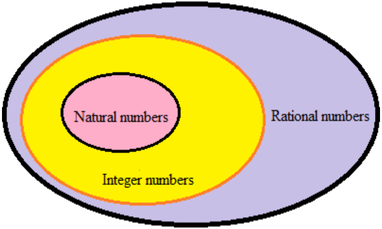
- $\pi =$ 3.14159265358979323846264338327950... .
- You cannot write down a simple fraction that equals π.
- So, it is an irrational number.
- The popular approximation of $\dfrac{22}{7} = 3.1428571428571\ldots$ is close but not accurate.

$\sqrt{2}$ is an irrational number

- $\sqrt{2}$=1.4142135623730950...(etc).
- It cannot be written as a ratio of two numbers.
- So, it is an irrational number.

Remark

- Every integer can be written as a fraction with denominator 1.
- So, the set of all rational numbers includes the integers.
- Eg: -7 can be written -7/1.
- Eg: 3 can be written 3/1.



Real numbers ($\mathbb{R}$)

- The real numbers include all of the measuring numbers.
- The set of real numbers includes all integers, all rational and the all irrational numbers.
- Every rational number is also a real number.
- It is not the case, however, that every real number is rational.

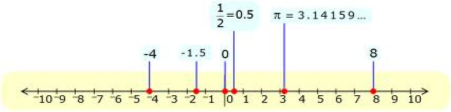
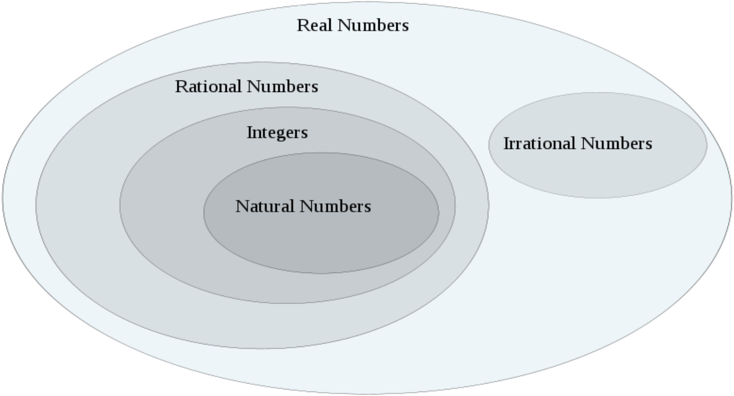


Diagram of number system



Examples

Classify following numbers according to number type.

(i) 0.45

(ii) 10

(iii) 5/3

(iv) $1\frac{2}{3}$

(v) -9/3

Solutions
(i) 0.45

- It can be written as a fraction: $45/100 \Rightarrow 9/20$.
- This fraction does not reduce to a whole number.
- So it is not an integer or a natural.
- Since it is a ratio of two integers, it is **rational number**.
- Every rational number is a real number.
- Therefore it is also a **real number**.

Solutions
(ii) 10

- This is a counting number.
- So it is a **natural number**.
- Every natural number is an integer.
- So it is an **integer**.
- This can be written as $10/1$. Therefore it is **rational number**.
- Every rational number is a real number.
- So it is also a **real number**.

Solutions
(iii) 5/3

- This is a fraction.
- So it is a **rational number**.
- It is also a **real number**.

Solutions

(iv) $1\frac{2}{3}$

- This can also be written as $5/3$.
- So it is **rational** and **real**.

Solutions
(v) -9/3

- This is a fraction, but notice that it reduces to -3.
- So this may also count as an integer.
- Therefore -9/3 is an **integer**, a **rational** and a **real**.

Why do we need another number type?

Equation 1	Equation 2
$x^2 - 1 = 0$ $x^2 = 1$	$x^2 + 1 = 0$ $x^2 = -1$

- Equation 1 has solutions because the number 1 has two square roots, 1 and -1.
- Equation 2 has no solutions because -1 does not have a square root.

Why do we need another number type?
Cont...

- In other words, there is no number such that if we multiply it by itself we get -1.
- If Equation 2 is to be given solutions, then we must create a square root of -1.

Complex numbers ($\mathbb{C}$)

- A complex number is one of the form $a + bi$, where a and b are real numbers.
- a is called the real part of the complex number, and b is called the imaginary part.
- i is a symbol with the property that $i^2 = -1$.

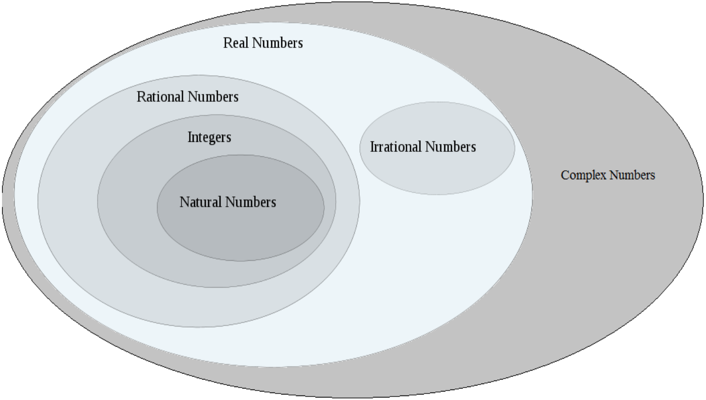
$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \\ x^2 &= i^2 \\ x &= \pm i \end{aligned}$$

Remark 1

Every real number is a complex number

- The real number 5 is equal to the complex number $5 + 0i$.
- The real number -9.12 is equal to the complex number $-9.12 + 0i$.

Diagram of whole number system



Cojugate of a complex number

- A complex number z is a number of the form $z = x + yi$.
- Its conjugate $\overline{z}$ is a number of the form $\overline{z} = x - yi$.
- The complex number z and its conjugate $\overline{z}$ have the same real part.
- The sign of the imaginary part of the conjugate complex number is reversed.

Find the conjugates of the following complex numbers.

(i) $4 + 3i$

(ii) $5 - 2i$

(iii) $7i + 2$

(iv) $-8i$

(v) 15

(vi) $3 + \sqrt{5}$

Cojugate of a complex number
Examples ⇒ Solutions

(i) $4 - 3i$

(ii) $5 + 2i$

(iii) $-7i + 2$

(iv) $8i$

(v) 15

(vi) $3 + \sqrt{5}$

Arithmatic operations on integers

If both a and b are integers (i.e $a, b \in \mathbb{Z}$), then

1. $a + b \in \mathbb{Z}$.
2. $a - b \in \mathbb{Z}$.
3. $a \times b \in \mathbb{Z}$.

Arithmatic operations on real numbers

If both a and b are real numbers(i.e $a, b \in \mathbb{R}$), then

1. $a + b \in \mathbb{R}$.
2. $a - b \in \mathbb{R}$.
3. $a \times b \in \mathbb{R}$.
4. $p/q \in \mathbb{R}$ when $q \neq 0$.

Arithmatic operations on complex numbers

If both z_1 and z_2 are complex numbers(i.e $z_1, z_2 \in \mathbb{C}$), then

1. $z_1 + z_2 \in \mathbb{C}$.
2. $z_1 - z_2 \in \mathbb{C}$.
3. $z_1 \times z_2 \in \mathbb{C}$.
4. $z_1 / z_2 \in \mathbb{C}$.

Arithmatic operations on complex numbers
Proof [1]

Let $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$.

$$\begin{aligned} z_1 + z_2 &= x_1 + y_1 i + x_2 + y_2 i \\ &= (x_1 + x_2) + (y_1 + y_2) i \in \mathbb{C} \end{aligned}$$

Proof [2]

Let $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$.

$$\begin{aligned} z_1 - z_2 &= (x_1 + y_1 i) - (x_2 + y_2 i) \\ &= (x_1 - x_2) + (y_1 - y_2) i \in \mathbb{C} \end{aligned}$$

Arithmatic operations on complex numbers

Proof [3]

Let $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$.

$$
\begin{aligned}
z_1 \times z_2 &= (x_1 + y_1 i) \times (x_2 + y_2 i) \\
&= x_1 x_2 + x_1 y_2 i + y_1 x_2 i + y_1 y_2 i^2 \\
&= x_1 x_2 + x_1 y_2 i + y_1 x_2 i + y_1 y_2 (-1) \\
&= (x_1 x_2 - y_1 y_2) + (x_1 y_2 + y_1 x_2) i \in \mathbb{C}
\end{aligned}
$$

Arithmatic operations on complex numbers
Proof [4]

Let $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$.

$$
\begin{aligned}
\frac{z_1}{z_2} &= \frac{x_1 + y_1 i}{x_2 + y_2 i} \\
&= \frac{(x_1 + y_1 i)}{(x_2 + y_2 i)} \frac{(x_2 - y_2 i)}{(x_2 - y_2 i)} \\
&= \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} \right) + \left(\frac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2} \right) i \in \mathbb{C}
\end{aligned}
$$

Let $z_1 = 5 + 2i$ and $z_2 = 4 - 3i$. Find followings,

(i) $\overline{z_1}$.

(ii) $\overline{z_2}$.

(iii) $z_1 + z_2$.

(iv) $z_1 - z_2$.

(v) $z_1 . z_2$.

(vi) z_1 / z_2.

Remark 2

- Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.
- That is, $a + bi = c + di$ if and only if $a = c$, and $b = d$.

Find the values of x and y.

(i) $5 + 7i = x + yi$

(ii) $-4 + 9i = 2x + 3yi$

(iii) $9 + 8i = -3 + 4i + 3xi + 2y$

(iv) $x + yi = 5$

(v) $x + yi = (3 + i)(2 - 3i)$

(vi) $\dfrac{x + yi}{2 + i} = 5 - i$

Summary of different type of numbers

Natural	(0), 1, 2, 3, 4, 5, 6, 7, ..., n
Integers	-n, ..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ..., n
Positive integers	1, 2, 3, 4, 5, ..., n
Negative integers	-1, -2, -3, -4, -5, ..., -n
Rational	a/b where a and b are integers and b is not zero
Real	all of the measuring numbers
Complex	a + bi where a and b are real numbers and i is the square root of -1

Thank you