# AN ANALYSIS OF DIRECT RESISTANCE HEATING OF A CIRCULAR SHEET METAL USING FINITE DIFFERENCE TECHNIQUES

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## Abstract

*This paper presents the analytical results of a study on direct resistance heating of a circular metal sheet with uniform thickness. The electrical current distribution, heat generation and temperature distribution are found when a circular sheet blank is heated by passing an electrical current through a diameter. It is shown that the elecrical current density of the plane sheet satisfies the continuity equation while the electrical potential satisfies the Laplace equation if there is no variation of physical properties of the material. The power balance equation which describes the temperature of the circular plate is obtained. The electrical potential, current density, generated heat, and temperature at each grid point are found and power balance equation is solved using finite difference technique. The results show that the current density, heat generation, and temperature near the diameter is higher than other points of the plate.*

### *List of Symbols*

$a$ - *Radius of the circular sheet, m*

$a_0$ - *Radius of the electrodes, m*

$b$ - *Thickness of the sheet, m*

$C$ - *Specific Heat,* $J\ kg^{-1}K^{-1}$

$d$ - *Density of the material,* $kg\ m^{-3}$

$F$ - *Rate of heat generation,* $W\ m^{-3}$

$h$ - *Convective heat transfer coefficient,* $W m^{-2}K^{-1}$

$\underline{J}$ - *Current density,* $Am^{-2}$

$L$ - *Lorenz constant,* $V^2K^{-2}$

$s$ - *Time step, s*

$T$ - *Temperature, K*

$t$ - *Time, s*

$\alpha$ - *Temperature coefficient of resistivity,* $K^{-1}$
$\in$ - *Total emissivity*
$\lambda$ - *Thermal conductivity,* $Wm^{-1} K^{-1}$
$\nu$ - *Charge density,* $Cm^{-3}$
$\rho$ - *Electrical resistivity,* $\Omega\, m$
$\rho_0$ - *Electrical resistivity at room temperature,* $\Omega\, m$
$\sigma$ - *Stefan-Boltzmann constant,* $Wm^{-2}K^{-4}$
$\phi$ - *Electrical potential, V*

## 1. Introduction

### 1.1 Direct Resistance heating

Direct resistance heating is accomplished by the dissipation of an electrical energy in a material. Alternating or direct current is passed between two electrodes through the material. Resistance losses are dissipated as heat, causing the material to heat. Direct resistance heating offers manufacturers precise control and directed heat for applications such as preheating billets for forging, producing unique hardening patterns on metals, continous annealing of wires (Kraft, 1994, 289), selectively heating forging dies, maintaining heat sources at a constant temperature (Reindl, 1992, 180) and maintaining solutions at constant temperature. Direct resistance heating works only for electrically conductive workpieces and it may be the simplest and most economical method for heat treating workpieces. By generating heat within the workpiece rather than in a furnace, direct resistance heating offers number of benefits over fuel-fired furnaces such as (1) rapid startup, (2) higher heating rates, (3) more comfortable working environment because heating takes place only in the workpiece (4) ease of autómation, (5) no pollution of atmosphere, (6) cleaner working environment, (7) lower maintenance.

The major metal working applications of direct resistance heating are heating prior to forming, heat treating, and seam welding. Minimum equipment requirements for a direct resistance heating system are the current input electrodes and their attachment to the product, a power supply and power regulation equipment, product handling system and a control system. Automated systems require load handling equipment, and some systems include water-cooling to prolong electrode life. The power electrodes are critical since they determine how much current can be passed to the workpiece, heating rate depends on the current flow. They must also be low in resistance to reduce heating. Contacts are often water cooled to prolong their life. Contact design is generally done by the equipment manufacturer. The power supply is often equipped with a voltage step-down transformer.

### 1.2 Present Problem

Direct resistant heating involves passing a current through electrically conductive materials and it has a high efficiency compared to other heating methods (Karunasena et al, 1981, 476-481). They have discussed the problems associated with the use of direct resistance heating by referring to investigations on the heating characteristics of short, rectangular sheet blanks. The objective of present study is to obtain heating and current transmission of a circular metal sheet blank with uniform thickness when it is heated by passing the electrical current through some of the diameters of the plate in a staggered pattern.

In order to obtain temperature distribution of the circular plate, we derive the power balance equation for the plate when it is heated by passing an electrical current through a diameter. This equation describes the temperature variation of the plate. Before solving this equation, we have to find the rate of heat generation per unit volume of the plate. As the heat generation depends on the current distribution it is necessary to find electrical potential distribution of the plate to obtain the current distribution.

In section 2, we describe the problem and obtain the power balance equation. Then we obtain an expression for the rate of heat generation of the plate. We show that the current density function of a plate satisfies the continuity equation. We also show that the electrical potential of a plate with uniform thickness satisfies the Laplace equation if there is no variation of the heating current and physical properties of the material.

In section 3, we find the electrical potential distribution using finite difference technique. We use these results and the relationship between electrical current density and electrical potential to obtain the current density distribution of the plate. Then we find the rate of heat generation.

In section 4, the temperature distribution of the plate is found by solving the power balance equation.

## 2. Formulation of the Problem

Figure 1($a$) shows the plan view of the circular sheet metal workpiece. Two cylindrical electrodes are pressed on to the diametrically opposite ends of the circle. A voltage is applied at the electrode and a large current is passed through the workpiece. Major portion of the current flows along the diameter because it is the path with least resistance. Remaining part of the current flows through both side of the diameter depending on the length of the path (and hence the resistance).

This uneven current flow causes uneven heating. To reduce this unevenness current should be passed through some other diameters of the circular sheet plate. For example, we can consider four diameters spaced at $45^0$ angles and the current can be passed in a staggered pattern varying the contact points of celectrodes as shown in Figure 1.

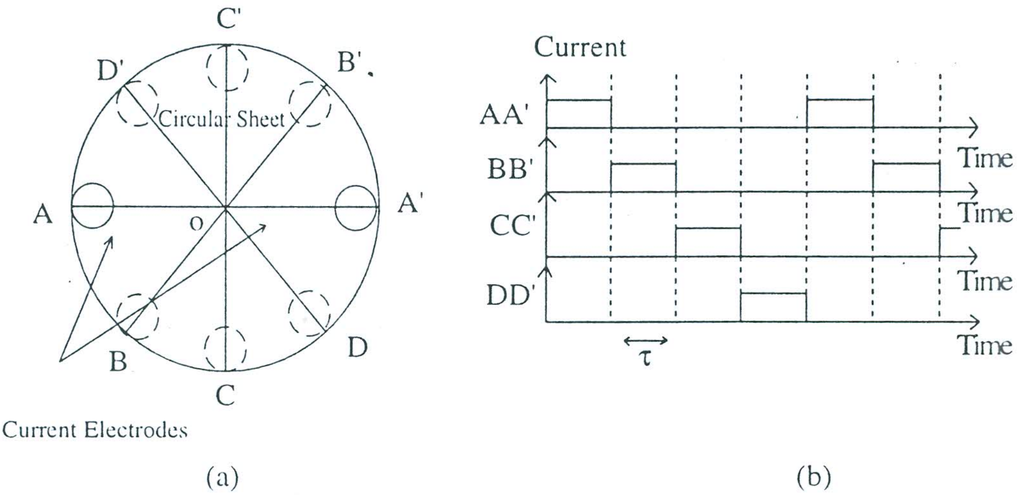


Figure 1: Placing of electrodes

### 2.1 Power Balance Equation

Let us derive the power balance equation which describes the relationship between temperature and the generated heat. Establish a polar coordinate system on the plate as shown in Figure 2 and consider a small element at $X \equiv (r, \theta)$ of the plate. The quantity of heat entering through the surface at $XX'$ is given by the Fourier's law (Sears et al, 1982, 291; James et al, 1977, 480), of heat conduction as

$$dq_1 = -br\lambda\Delta\theta\left(\frac{\partial T}{\partial r}\right)_r dt \tag{2.1}$$

where $\lambda$ is the coefficient of thermal conductivity, b is the thickness of the plate, and $\left(\frac{\partial T}{\partial r}\right)_r$ is the average temperature gradient over the surface at $XX'$. Similarly

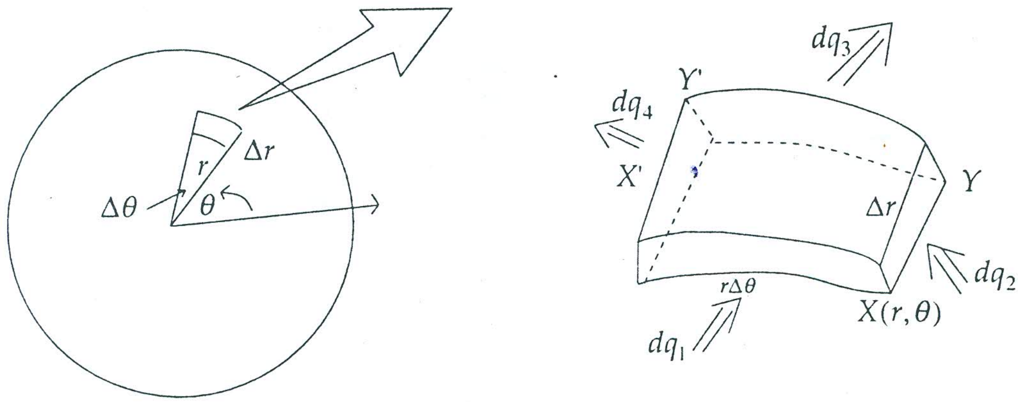


Figure 2: Heat conduction

$$dq_2 = \frac{-b\lambda\Delta r}{r}\left(\frac{\partial T}{\partial\theta}\right)_{\theta} dt, \qquad (2.2)$$

$$\text{and } dq_3 = -b\lambda(r+\Delta r)\Delta\theta\left(\frac{\partial T}{\partial r}\right)_{r+\Delta r} dt, \qquad (2.3)$$

$$dq_4 = \frac{-b\lambda\Delta r}{r}\left(\frac{\partial T}{\partial\theta}\right)_{\theta+\Delta\theta} dt, \qquad (2.4)$$

$dQ=(dq_1-dq_3)+(dq_2-dq_4)$ is the total heat gained by the element in time $dt$ and hence the rate of net heat gained by the element by conduction is

$$\frac{dQ}{dt} = br\lambda\Delta\theta\left[\left(\frac{\partial T}{\partial r}\right)_{r+\Delta r} - \left(\frac{\partial T}{\partial r}\right)_{r}\right] + b\lambda\Delta r\Delta\theta\left(\frac{\partial T}{\partial r}\right)_{r+\Delta r} + \frac{b\lambda\Delta r}{r}\left[\left(\frac{\partial T}{\partial\theta}\right)_{\theta+\Delta\theta} - \left(\frac{\partial T}{\partial\theta}\right)_{\theta}\right] \qquad (2.5)$$

The rate of heat generated in the element is $br\ F(r,\theta)\Delta r\Delta\theta$; where F(r,θ) is the rate of heat generation per unit volume.

The rate of heat energy storage in the element is $brd\ C\Delta r\Delta\theta\frac{\partial T}{\partial t}$;

where $d$ is the mass density of the plate, $C$ is the specific heat of the material (Sears et al, 1982, 276-278; James et al, 1977, 480).

According to *Stefan-Boltzmann Law* (Sears et al, 1982, 298; James et al, 1977, 483) the rate of heat radiated from the surfaces is $\in \sigma r\Delta r\Delta\theta(T^4 - T_0^4)$; where $\in$ is total emissivity, $\sigma$ is Stafan-Boltzmann constant and $T_0$ is room temperature.

The rate of heat loss by convection is $hr\Delta r\Delta\theta(T - T_0)$; where $h$ is convective heat transfer coefficient (Sears et al, 1982, 294-295).

According to the law of conservation of thermal energy, the rate of heat gained by the element and the rate of heat generated in the element must be equal to the rate of heat loss, and the rate of heat energy storage in the element. Applying this law, we get

$$\frac{dQ}{dt} + br\Delta r\Delta\theta F(r,\theta) = \in \sigma r\Delta r\Delta\theta(T^4 - T_0^4)$$

$$+ hr\Delta r\Delta\theta(T - T_0) + bdrC\Delta r\Delta\theta\frac{\partial T}{\partial t}. \quad (2.6)$$

Substituting for $\frac{dQ}{dt}$ and dividing by $r\Delta r\Delta\theta$ and taking limits $\Delta r \to 0$ and $\Delta\theta \to 0$, we get,

$$\lim_{\Delta r \to 0}\left\{\frac{(\lambda\frac{\partial T}{\partial r})_{r+\Delta r} - (\lambda\frac{\partial T}{\partial r})_r}{\Delta r} + \frac{\lambda}{r}(\frac{\partial T}{\partial r})_{r+\Delta r}\right\}$$

$$+ \lim_{\Delta\theta \to 0}\frac{(\lambda\frac{\partial T}{\partial\theta})_{\theta+\Delta\theta} - (\lambda\frac{\partial T}{\partial\theta})_\theta}{r^2\Delta\theta} + F(r,\theta) \quad (2.7)$$

$$= \frac{\in\sigma}{b}(T^4 - T_0^4) + \frac{h}{b}(T - T_0) + dC\frac{\partial T}{\partial t}$$

i.e.

$$\frac{\partial}{\partial r}(\lambda\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial\theta}(\lambda\frac{\partial T}{\partial\theta}) + F(r,\theta)$$

$$= \frac{\in\sigma}{b}(T^4 - T_0^4) + \frac{h}{b}(T - T_0) + dC\frac{\partial T}{\partial t} \quad (2.8)$$

This is the power balance equation. To solve this equation we have to find $F(r,\theta)$. Similar equation for a short, rectangular sheet blank was used by Karunasena (Karunasena, 1977, 24).

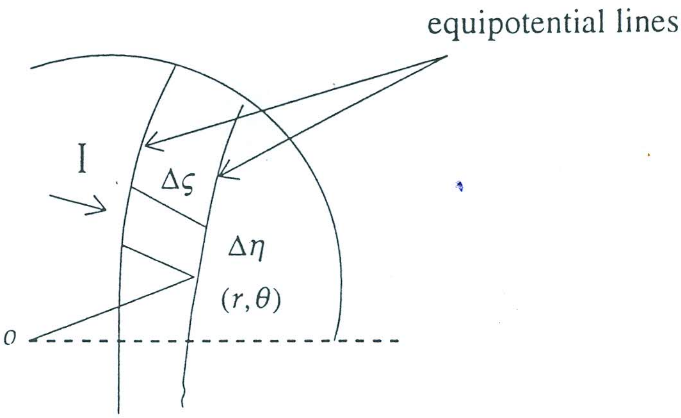


Figure 3: Small element between two equipotential lines

### 2.1.1 Heat Generation

Let $\underline{J}(r,\theta)$ be the current density at the point $(r,\theta)$, and $\rho$ be the electrical resistivity of the material. Consider a very small element which has an area $\Delta\eta\Delta\zeta$ as shown in Figure 3. Let us use the well-known relationship, $F = I^2 R$; where $F$ is generated energy, I is current and $R$ is resistance (Borowitz & Beiser, 1971; Sears et al, 1982, 500).

Therefore the total energy generated in the element $b\,\Delta\eta\Delta\zeta$ is given by

$$b\,\Delta\eta\Delta\zeta F(r,\theta) = (|\,\underline{J}\,|\, b\Delta\eta)^2 \rho \frac{\Delta\zeta}{b\Delta\eta} \tag{2.9}$$

i.e

$$F(r,\theta) = |\,\underline{J}\,|^2\ \rho. \tag{2.10}$$

### 2.1.2 Continuity Equation

Now we have to find $\underline{J}$ in order to find $F(r,\theta)$. Let us derive the continuity equation which describes $\underline{J}$ (Panofsky & Phillips, 1978, 118-119). Let $\underline{J} = (J_x, J_y)$ be the current density of the lamina at $(x, y)$, see Figure 4 and let $b$ be the thickness of the lamina. Then the charge, per second, which enters through the surfaces at $AB$ and $AD$ are $J_x b\Delta y$ and $J_x b\Delta x$ respectively, and the charge, per second, which vacates through the surfaces at $CD$ and $BC$ are $(J_x + \frac{\partial J_x}{\partial x}\Delta x)b\Delta y$ and $(J_y + \frac{\partial J_y}{\partial y}\Delta y)b\Delta x$ respectively. Therefore the change of charge, per second, of the volume $b\Delta x\Delta y$ is $-\left(\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}\right)b\Delta x\Delta y$ . This is equal to the time rate of change of charge, $\frac{\partial}{\partial t}(bv\Delta x\Delta y)$; where v is the charge density at $(x, y)$. i.e.

$$-\left(\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}\right)b\Delta x\Delta y=\frac{\partial}{\partial t}(bv\Delta x\Delta y). \tag{2.11}$$

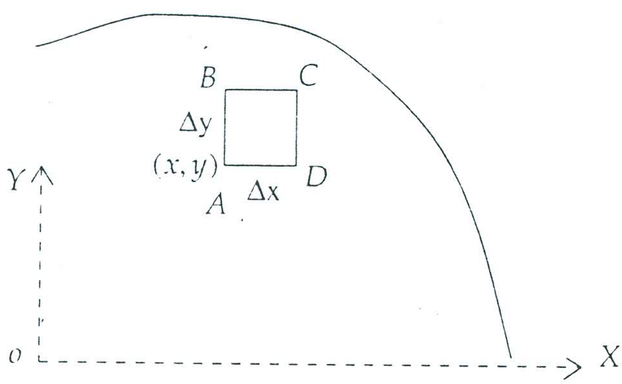


Figure 4: Small element in cartesian coordinates

Since the volume $b\Delta x\Delta y$ is arbitrary and does not depend on t, we get

$$-\left(\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}\right)=\frac{\partial v}{\partial t}, \tag{2.12}$$

which gives

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right).(J_x, J_y)+\frac{\partial v}{\partial t}=0 \tag{2.13}$$

$$i.e. \qquad \underline{\nabla}.\underline{J}+\frac{\partial v}{\partial t}=0. \tag{2.14}$$

If $v$ does not depend on time then we get

$$\underline{\nabla}.\underline{J}=0. \tag{2.15}$$

Since the current density satisfies the relationship $\underline{J}=-\frac{1}{\rho}\underline{\nabla}\phi$ where $\phi$ is the electrical potential (Sears, 1979, 91), we get

$$\underline{\nabla}.\left(\frac{1}{\rho}\underline{\nabla}\phi\right)=0. \tag{2.16}$$

If electrical resistivity is a constant then $\phi$ satisfies the Laplace equation,

$$\nabla^2\phi=0. \tag{2.17}$$

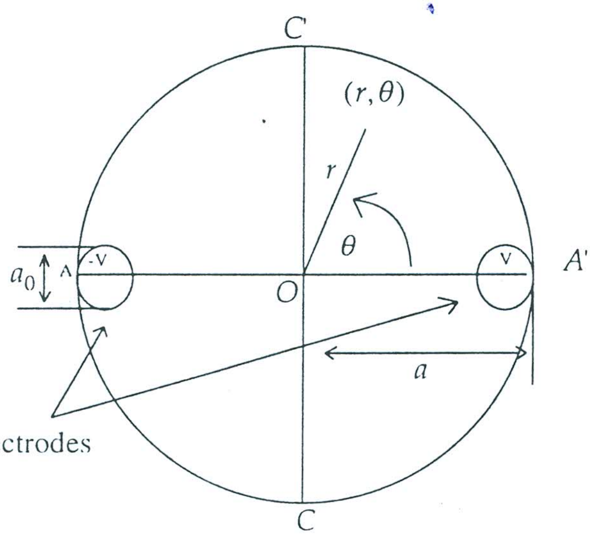


Figure 5: Dimensions of the plate and electrodes

## 3. Electical Potential and Current Density Distributions

Let electric potentials at A and A′ be $-V$ and $V$ respectively. Then the potential difference is $2V$. According to the symmetrical shape of the plate, electrical potential on $CC'$ is zero when there is no temperature variation of the plate. Since there is no current flow through the boundary, we have the boundary condition

$$\left(\frac{\partial\phi}{\partial r}\right)_{r=a}=0; \tag{3.1}$$

where a is the radius of the circular plate. Let $a_0$ be the radius of a electrode and let $\phi=-V$ at the electrode with center $(x_1, y_1)$ and $\phi=V$ at the electrode with center $(x_2, y_2)$. Then

$$\begin{aligned}\phi(r,\phi)&=-V \text{ if } \quad (r\cos\theta-x_1)^2+(r\sin\theta-y_1)^2\le a_0^2,\\ \phi(r,\phi)&=V \text{ if } \quad (r\cos\theta-x_2)^2+(r\sin\theta-y_2)^2\le a_0^2.\end{aligned} \tag{3.2}$$

Therefore we have two other boundary conditions.

## 3.1 Variation of Physical Properties with Temperature

### 3.1.1 Electrical Resistivity

Electrical resistivity of a metal changes with temperature. Let $\rho_0$ be the electrical resistivity at room temperature ($T_0$ = 300K)and $\alpha$ be the temperature coefficient of resistivity. Then $\rho$ can be written as, (Sears, 1982, 487),

| Temperature (°C) | Specific Heat (J(kg$^{-1}$)(°C$^{-1}$) |
|---|---|
| 0 | 515 |
| 25 | 518 |
| 200 | 540 |
| 400 | 565 |

Table 1: Specific heat of Stainless Steel

$$\rho = \rho_0(1+\alpha(T-T_0)). \tag{3.3}$$

### 3.1.2 Thermal Conductivity

According to *Wiedemann-Franz-Lorenz Law* (Gray et al, 1972, 4-148) the relationship among thermal conductivity ($\lambda$), electrical resistivity ($\rho$) and temperature ($T$) we can write as

$$\lambda = \frac{LT}{\rho}; \tag{3.4}$$

where $L$ is Lorenz constant. Therefore we can write

$$\lambda = \frac{LT}{\rho_0(1+\alpha(T-T_0))} \tag{3.5}$$

### 3.1.3 Specific Heat

Specific heat ($C$) of a metal also changes with temperature. Some values of Specific heat of stainless steel at several temperature values were used by Karunasena (Karunasena, 1977, 26). Table 1 shows those values. Using these values we can obtain the following relationship between $C$ and $T$.

$$C = 0.122T + 482.24. \quad (3.6)$$

## 3.2 Electrical Potential and Current Density

Expanding the equation (2.16), we get

$$\nabla^2\phi - \frac{1}{\rho}\underline{\nabla}\,\rho.\underline{\nabla}\,\phi = 0. \quad (3.7)$$

Substituting from equation (3.3) and expanding this equation in polar coordinates, we get

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\alpha}{1+\alpha(T-T_0)}\left(\frac{\partial T}{\partial r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial T}{\partial\theta}\frac{\partial\phi}{\partial\theta}\right) = 0 \quad (3.8)$$

Now we have to solve this equation with the above mentioned boundary conditions. After solving this equation we can find the current distribution using the relationship $\underline{J} = -\frac{1}{\rho}\underline{\nabla}\phi$. This equation can be written in polar coordinates as

$$|\underline{J}| = \frac{1}{\rho}\left(\left(\frac{\partial\phi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\phi}{\partial\theta}\right)^2\right)^{\frac{1}{2}} \quad (3.9)$$

## 3.3 Finite Difference Method

It is impossible to solve the above equations analytically. Therefore let us use the finite difference method, (Jain et al, 1994, 113-115; Smith, 1985, 245-247), to solve these equations. The following figure shows a grid superimposed upon the circular plate.

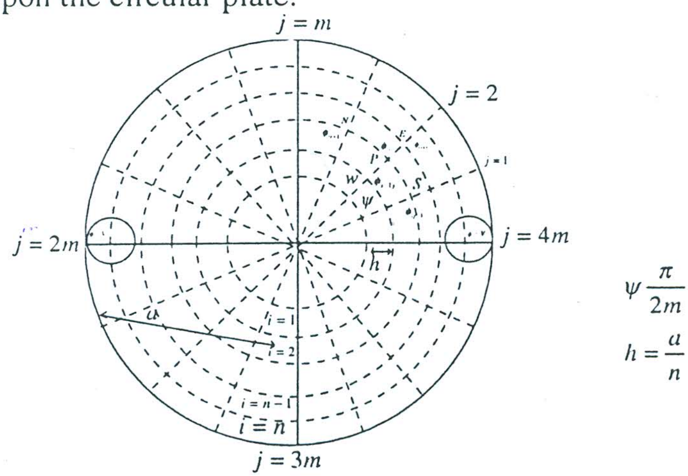


Figure 6: Typical mesh for scheme 3.10

Let us consider the region $0 \leq r \leq a, 0 \leq \theta \leq 2\pi$. Let $P \equiv (r,\theta) = (hi, \psi j), E \equiv (h(i+1), \psi j), W \equiv (h(i-1), \psi j), N \equiv (hi, \psi(j+1))$ and $S \equiv (hi, \psi(j-1))$, and potentials at *P, E, W, N* and *S* be $\phi_{ij}, \phi_{i+1,j}, \phi_{i-1,j}, \phi_{i,j+1}$ and $\phi_{i,j-1}$ respectively for *i = 0,1,2, ........n, j=1,2,3, ........., 4m.*

Also let s be the time step and *l* be the number of time steps such that the time interval $(o,t) \equiv (0, sl)$ and let $T^{l}_{i,j}$ for *i = 0,1,2,........n, j=1,2,3, ........., 4m* be the temperature at $P \equiv (hi, \psi j)$ at time *sl*.

Thus, the equation (3.8) can be written using five point difference scheme as

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{h^2} + \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2h^2 i} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{h^2 i^2 \psi^2}$$
$$- \frac{\alpha}{1+\alpha(T^{l}_{i,j} - T^{0}_{i,j})} \left\{ \frac{(T^{l}_{i+1,j} - T^{l}_{i-1,j})}{2h} \frac{(\phi_{i+1,j} - \phi_{i-1,j})}{2h} \right. \tag{3.10}$$
$$\left. + \frac{(T^{l}_{i,j+1} - T^{l}_{i,j-1})}{2hi\psi} \frac{(\phi_{i,j+1} - \phi_{i,j-1})}{2hi\psi} \right\} = 0;$$

$$\phi_{i,j} = \frac{i^2\psi^2}{2+2i^2\psi^2} \left\{ \phi_{i+1,j}\left(1+\frac{1}{2i}\right) + \phi_{i-1,j}\left(1-\frac{1}{2i}\right) \right.$$
$$+ \frac{\phi_{i,j+1} + \phi_{i,j-1}}{i^2\psi^2} - \frac{\alpha(T^{l}_{i+1,j} - T^{l}_{i-1,j})(\phi_{i+1,j} - \phi_{i-1,j})}{4+4\alpha(T^{l}_{i,j} - T^{0}_{i,j})} \tag{3.11}$$
$$\left. - \frac{\alpha(T^{l}_{i,j+1} - T^{l}_{i,j-1})(\phi_{i,j+1} - \phi_{i,j-1})}{(4+4\alpha(T^{l}_{i,j} - T^{0}_{i,j}))i^2\psi^2} \right\}$$

Where *i = 1,2,...,n,j = 1,2,3,.....,4m.* Rearranging terms we get for *i=1,2,.....,n,j = 1,2,3,..,4m.*

To give a means of satisfying the boundary condition (3.1) we can write

$$\phi_{n,j} = \phi_{n-1,j} \tag{3.12}$$

for *j= 1,2,3....., 4m.*

Also, we have two conditions $\phi_{i,j} = \pm V$ at the electrodes. Let

$$\begin{aligned} x &= hi\cos(\psi j), \\ y &= hi\sin(\psi j), \\ e_1 &= (x-x_1)^2+(y-y_1)^2-a_0^2, \\ e_2 &= (x-x_2)^2+(y-y_2)^2-a_0^2. \end{aligned} \tag{3.13}$$

Then the above boundary conditions can be written as

$$\begin{aligned} \phi_{i,j} &= -V \quad \text{if} \quad e_1 \le 0, \text{and} \\ \phi_{i,j} &= V \quad \text{if} \quad e_2 \le 0 \end{aligned} \tag{3.14}$$

for $i = 1,2,........,n, j = 1,2,3,........,4m$.

To find electrical current using equation (3.9), we have to use following finite difference formulae.

$$J_{i,j} = \frac{1}{\rho_{i,j}}\left(\left(\frac{\phi_{i+1,j}-\phi_{i-1,j}}{2h}\right)^2+\frac{1}{h^2 i^2}\left(\frac{\phi_{i,j+1}-\phi_{i,j-1}}{2\psi}\right)^2\right)^{\frac{1}{2}}; \tag{3.15}$$

Where $J_{i,j}$ is current density at $P \equiv (hi, \psi j)$ and

$$\rho_{i,j} = \rho_0(1+\alpha(T_{i,j}^l - T_{i,j}^0)) \tag{3.16}$$

for $i = 0,1,2,........,n-1,\ j = 1,2,3,........,4m$.

Writing $F(r,\theta) = F(hi,\psi j) = F_{i,j}$ and using equation (2.10), we get

$$F_{i,j}^2 = J^2_{i,j}\rho_{i,j} \tag{3.17}$$

for $i = 0,1,2,........,n-1,\ j = 1,2,3,........,4m$.

## 4. Temperature Distribution

Now we are in a position to solve the power balance equation

$$\begin{aligned} &\frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\lambda\frac{\partial T}{\partial\theta}\right)+F(r,\theta) \\ &\qquad = \frac{\in\sigma}{b}(T^4-T_0^4)+\frac{h}{b}(T-T_0)+dC\frac{\partial T}{\partial t}. \end{aligned} \tag{4.1}$$

Let us assume that there are no heat losses by convection and radiation. Then the above equation can be written as

$$\frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\lambda\frac{\partial T}{\partial\theta}\right)+F(r,\theta)=dC\frac{\partial T}{\partial t}. \tag{4.2}$$

Substituting from equations (3.4), (3.5), (3.6), we get

$$\lambda\left\{\frac{\partial^2 T}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\right\}+\frac{L\rho_0}{\rho^2}(1-\alpha T_0)\left\{\left(\frac{\partial T}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial T}{\partial\theta}\right)^2\right\}$$
$$+F(r,\theta)=d(0.122T+482.24)\frac{\partial T}{\partial t}. \tag{4.3}$$

### 4.1 Finite Difference Method

Using five point finite difference scheme for derivatives of $r$ and $\theta$ and two level difference method for time derivative (Jain et al, 1994, 14-52; Smith, 1985, 75-77) in the equation (4.3) in polar coordinates and rearranging terms, we get

$$T^{l+1}{}_{i,j}=T^{l}{}_{i,j}+\frac{s}{dC_{i,j}}F_{i,j}$$
$$+\frac{s}{dC_{i,j}}\left\{\lambda_{i,j}\left(\frac{T^{l}{}_{i+1,j}-2T^{l}{}_{i,j}+T^{l}{}_{i-1,j}}{h^2}+\frac{T^{l}_{i,j+1}-2T^{l}{}_{i,j}+T^{l}_{i,j-1}}{h^2 i^2\psi^2}\right)\right.$$
$$\left.+\frac{L\rho_0}{\rho_{i,j}{}^2}(1-\alpha T^0_{i,j})\left(\frac{(T^{l}{}_{i+1,j}-T^{l}{}_{i-1,j})^2}{4h^2}+\frac{(T^{l}_{i,j+1}-T^{l}_{i,j-1})^2}{4h^2 i^2\psi^2}\right)\right\}; \tag{4.4}$$

where $C_{i,j}=(0.122T_{i,j}+482.24)$, for $i=1,2,\ldots.,\ n\text{-}1,\ \ j=1,2,\ldots\ldots..,4m,$ and $l=0,1,2,\ldots\ldots...,l_t,$ with $sl_t=t.$. We use $T^0_{i,j}=300K\ \forall i,j$ as initial conditions and $T^l_{n,j}=300K\ \forall j,l$ as boundary conditions.

## 5. Numerical Results

We use the following values to compute potentials, current densities, generated heat and temperature values at each grid point. The material properties for Stainless Steel are used for this study.

$a = 0.1m$

$a_0 = 0.005m$

$b = 0.2 \times 10^{-3} m$

$d = 7930 kgm^{-3}$

$L = 2.45 \times 10^{-8} V^2 K^{-2}$

$l_t = 1500$

$m = 45$

$n = 25$

$s = 1.575 \times 10^{-3} s$

$V = 0.5V$

$\alpha = 6.0 \times 10^{-4} K^{-1}$

$\rho_0 = 96.0 \times 10^{-8} \Omega m$

$Room\,temperature = 300K$

The potential, current density, generated heat and temperature distributions of the circular plate are plotted (see Figures 7-10). Figure 11 shows the variation of total current. These graphs are obtained by using the results of a FORTRAN program based on equations (3.11), (3.15), (3.17), and (4.4).

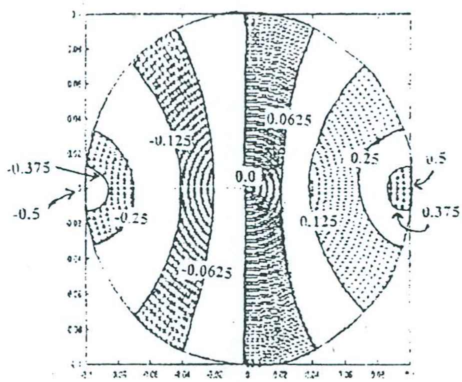


Figure 7: Potential distribution /V

## 6 Conclusions

This study has revealed how direct resistance heating influences temperature distribution of a circular sheet metal. Using the general laws of heat transfer and electricity, the power balance equation which describes the relationship between generated heat by an electric current and temperature of a circular sheet metal, has been derived. The other equation, (3.8), which describes the effect of the temperature on the potential distribution has been obtained using physical properties of metal and theories of current flow.

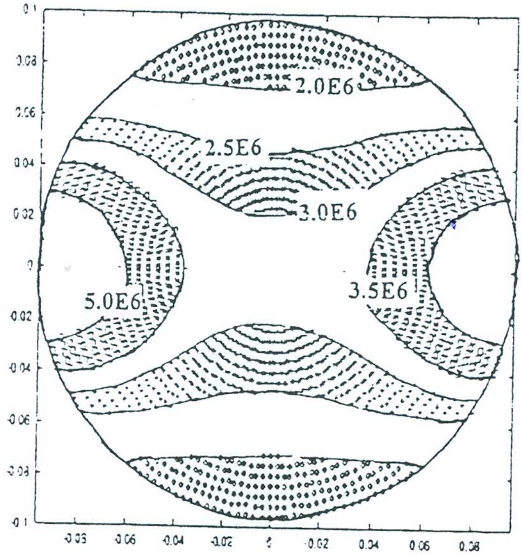


Figure 8: Current Density Distribution /Am$^{-2}$

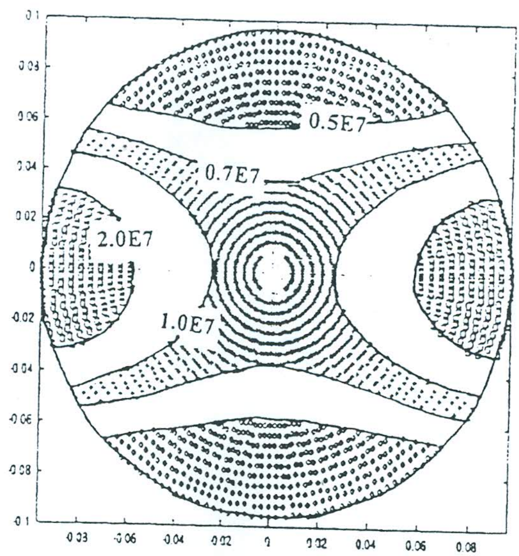


Figure 9: Distribution of Generated Heat /Wm$^{-3}$

To solve these equations we used the finite difference techniques. The results, see Figures 8-10 show that the current densities, generated heat, and temperature near the diameter and electrodes are higher than the other points of the plate. Since the current density at the electrodoes is higher than other points, the temperature is relatively high near the electrodes. Therefore, it is suggested to pass the current through other diameters of the plate in a staggered pattern, to get a uniform temperature distribution.

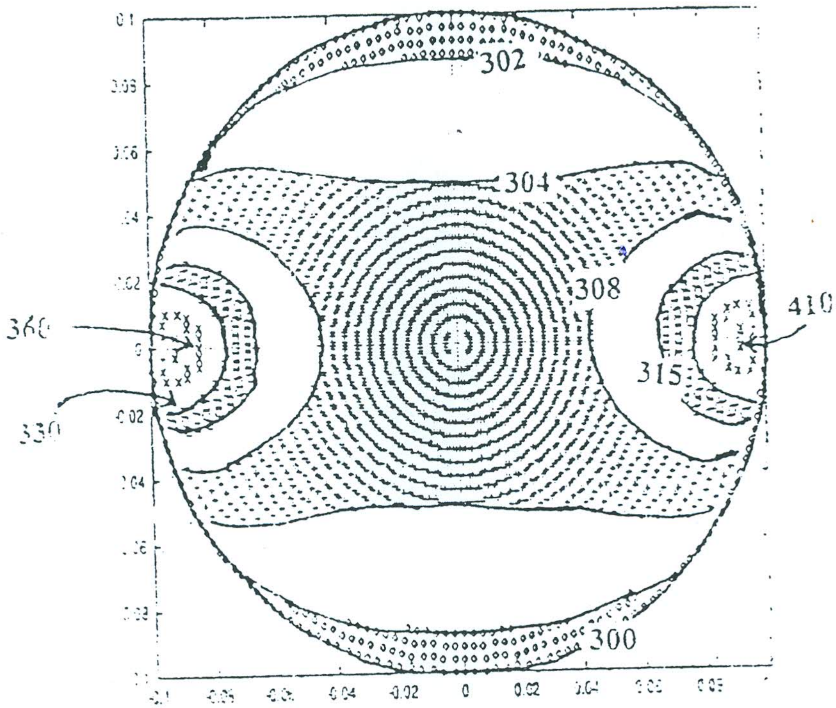


Figure 10: Temperature Distribution /K

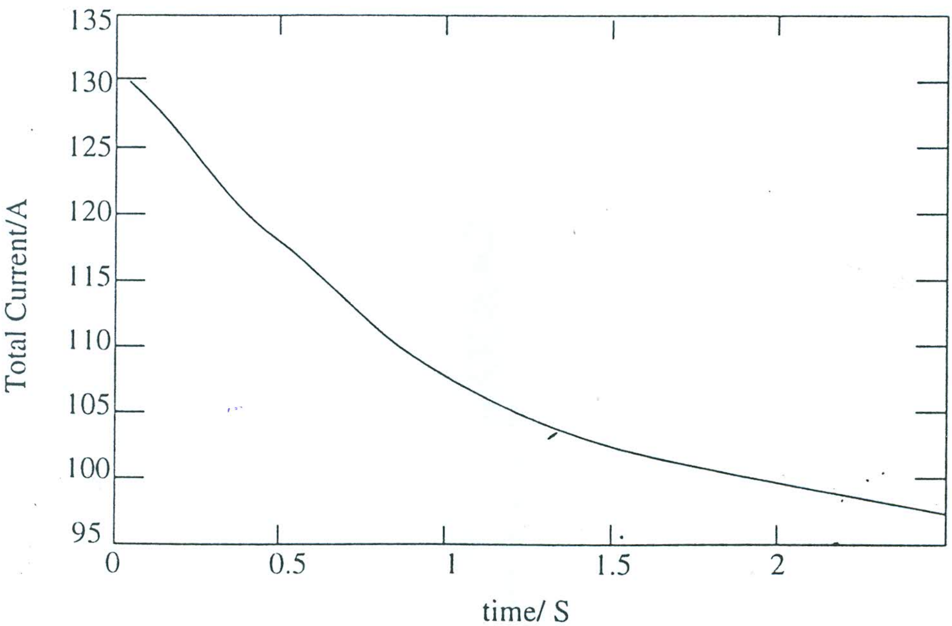


Figure 11: Total Current /A

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# PREFACE

Faculty of Graduate Studies of the University of Sri Jayewardenepura was established in 1996 and the First Annual Research Sessions were held on 27th March 1998. A selected collection of the papers presented in the sessions is compiled in this publication. Senior academic staff members of the university presented some of the papers while postgraduate students of the Faculty presented some others. Academic activities of the Faculty of Graduate Studies are conducted under six Boards of Studies. The Research Sessions were conducted in four technical sessions under (a) Social Sciences and Humanities (b) Life Sciences (c) Physical Sciences (d) Management and Commerce.

The objectives of the Research Sessions were mainly to provide a forum for presentation and discussion of the research findings of academic staff and research students and to create an awareness of the postgraduate studies carried out in the faculty among the academic community.

It is my hope that this publication will contribute to the knowledge and will provide reading material for students and researchers.

I am thankful to all the academic staff members who refereed the papers before acceptance for publication and the authors for helping me with editorial work. I am also grateful to Professor M.M.Karunanayake, Dean of the Faculty, for the help given in many ways to bring out this publication.

**Professor H. G.. Nandadasa**
Editor.